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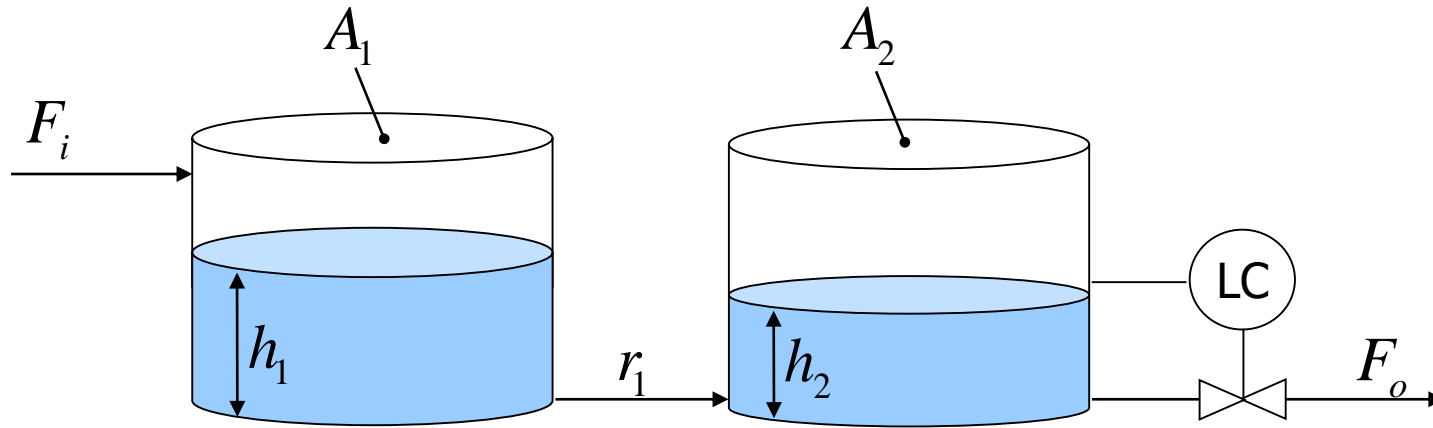
Dynamics and Control of Chemical Processes

Solution to Lab #3

Design of a control system



System representation



Data:

$$F_i = 9.4 \text{ m}^3/\text{s}$$

$$A_1 = 30 \text{ m}^2$$

$$A_2 = 50 \text{ m}^2$$

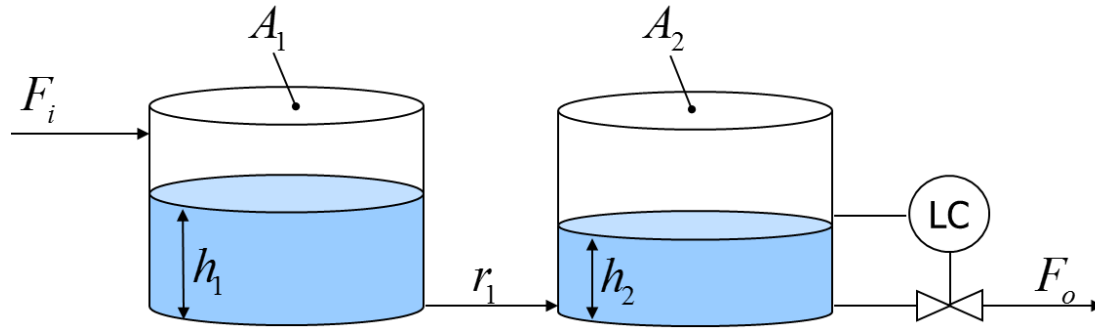
$$r_1 = 1.2 \text{ s}/\text{m}^2$$

I.C.:

$$h_2 = 6.6 \text{ m}$$

F_o changes linearly with the level: $F_o = 1.43 h_2$

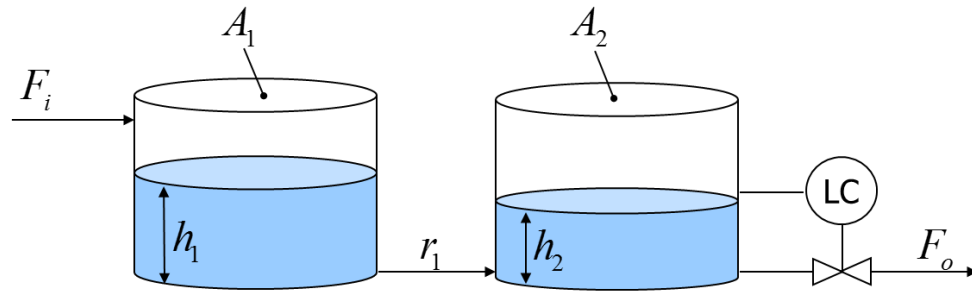
Tasks



1. Determination of the parameters of the proportional and the proportional-integral control systems by means of the **Cohen-Coon method** assuming there is a step disturbance on the inlet flowrate, such that it doubles. The task of the control system is to keep the steady state, i.e. to keep constant the level in the second tank. Assume that the system is in steady state when the step disturbance occurs.
2. Evaluate the system dynamics in closed loop.



Control philosophy



Controlled variable:

→ Level of the second tank

Manipulated variable:

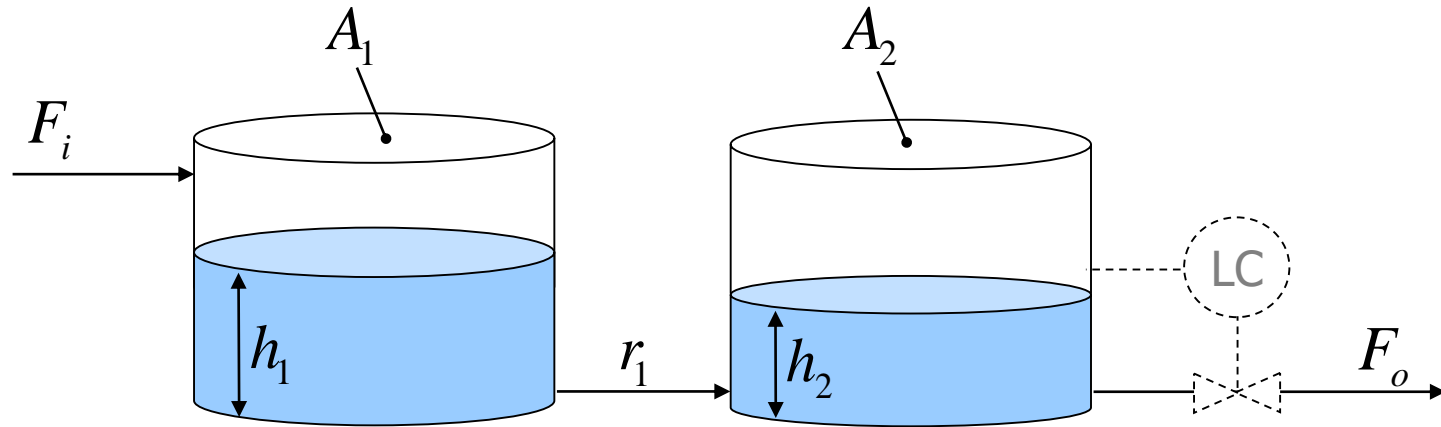
→ Valve opening =
= second tank outlet flowrate

Design of a control system with Cohen-Coon method

- A. Assign a step disturbance on the variable that could change during the plants operations (inlet flowrate). Find the dynamics of the controlled variable (liquid level in the second tank) in open-loop, i.e. without the control system.
- B. Find the characteristic parameters of the system:
 - A. gain of the system K ;
 - B. time delay: t_d ;
 - C. Characteristic time to reach the new steady state: τ .
- C. Find the control system parameters according to specific formulas.



A- Evaluation of the open-loop dynamics



Dynamics of the system

$$\begin{cases} A_1 \frac{dh_1}{dt} = F_i - \frac{h_1 - h_2}{r_1} \\ A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{r_1} - F_o \end{cases}$$

Open-loop outlet flow rate law:

$$F_o = 1.43 h_2$$

Evaluation of the steady state conditions

$$\left\{ \begin{array}{l} 0 = F_i^{(s)} - \frac{h_1^{(s)} - h_2^{(s)}}{r_1} \\ 0 = \frac{h_1^{(s)} - h_2^{(s)}}{r_1} - F_o^{(s)} \end{array} \right. \quad \rightarrow \quad \left\{ \begin{array}{l} F_o^{(s)} = F_i^{(s)} = 9.4 \text{ m}^3/\text{s} \\ h_1^{(s)} = r_1 F_i^{(s)} + h_2^{(s)} = 17.9 \text{ m} \end{array} \right.$$



MATLAB implementation

Main

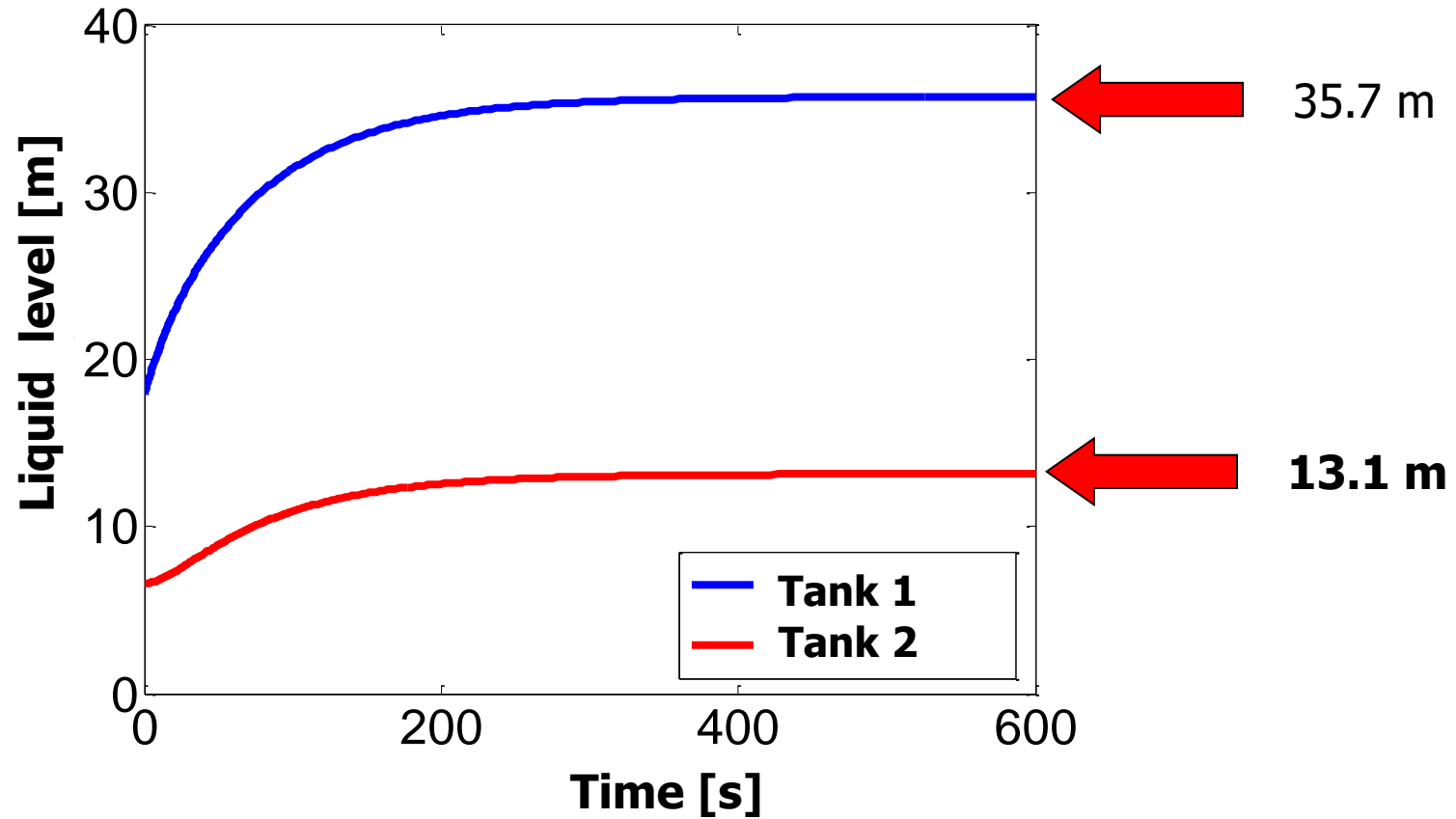
```
Fi_s = 9.4  
y0 = [17.9 6.6]  
tSpan = [0. 600.];  
[t,h] = ode45(@(t,y)Sisdif(t,y,A1,A2,r1,Fi_s),tSpan,y0,options);
```

Sisdif

```
function dy = Sisdif(t,y,A1,A2,r1,Fi_s)  
    h1 = y(1);  
    h2 = y(2);  
    Fi = Fi_s*2;  
  
    dy(1) = (Fi - (h1-h2)/r1) / A1;  
    dy(2) = ((h1-h2)/r1 - (1.43*h2)) / A2;
```



Open-loop dynamics



B - Controller parameters

Proportional control

$$K_c = \frac{1}{K} \frac{\tau}{t_d} \left(1 + \frac{t_d}{3\tau} \right)$$

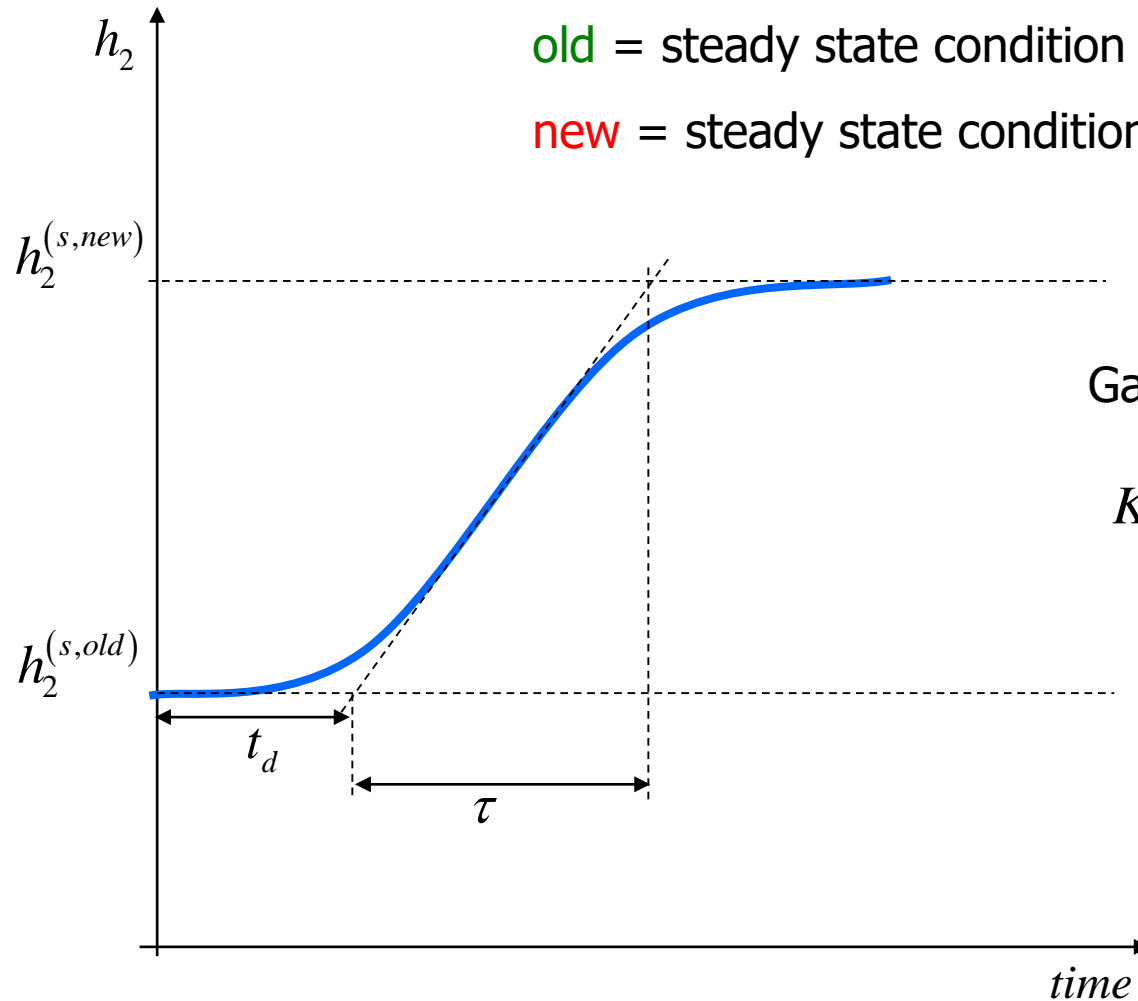
Proportional-integral control

$$K_c = \frac{1}{K} \frac{\tau}{t_d} \left(0.9 + \frac{t_d}{12\tau} \right)$$

$$\tau_I = t_d \left(\frac{30 + 3t_d/\tau}{9 + 20t_d/\tau} \right)$$



Characteristic parameters of the system



old = steady state condition before the disturbance

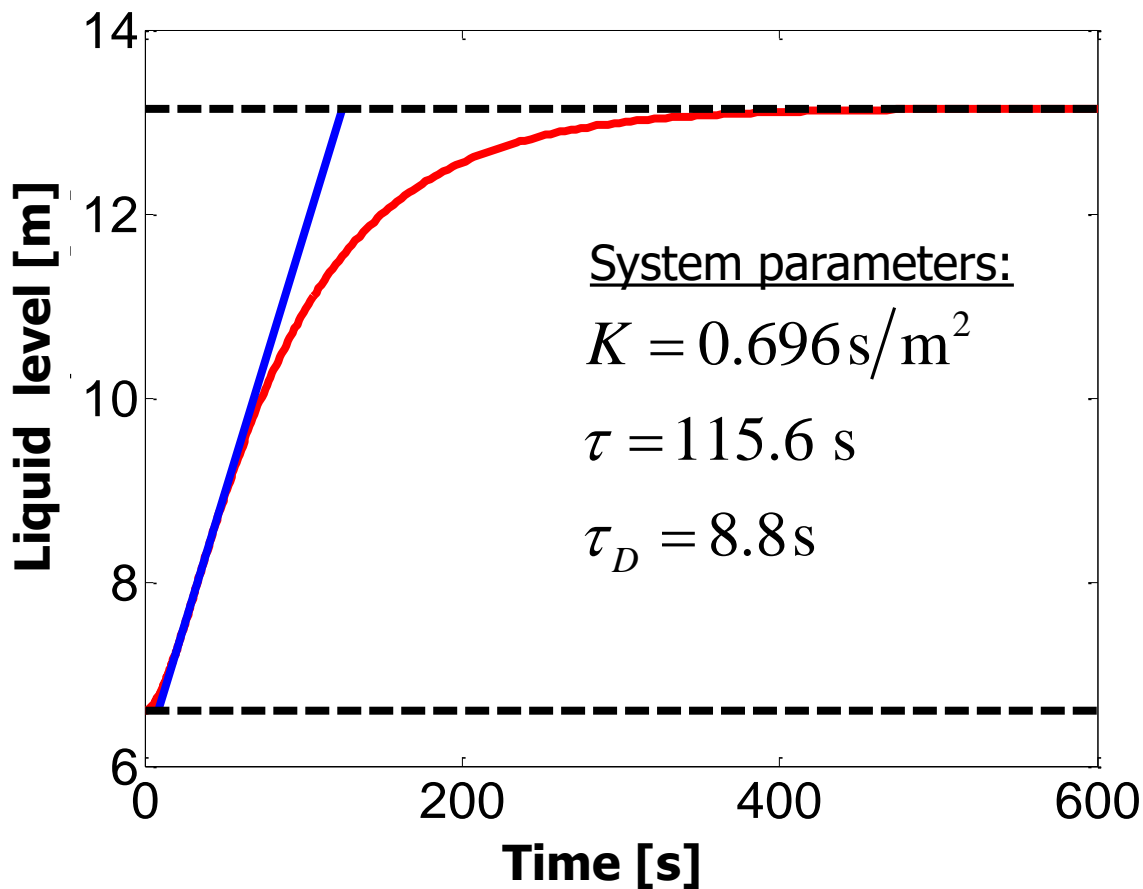
new = steady state condition after the disturbance

Gain of the system:

$$K = \frac{h_2^{(s,new)} - h_2^{(s,old)}}{F_i^{(new)} - F_i^{(old)}}$$



System dynamics for a step disturbance



C1- Proportional controller model

The general form of the model of the proportional control is:

$$c = c_s + K_c (y - y_{SP})$$

Where c_s is the bias, i.e. the value of the manipulated variable when the controlled variable is at the set-point. In this case a valve on the inlet flowrate is manipulated and the net result is the modification of the second tank outlet flowrate.

Assuming a linear relationship between the valve opening degree and the outlet flowrate, the controller model can be re-written as:

$$F = F_o^{(s)} + K_c (h_2 - h_2^{(s)})$$

The model of the system becomes:

$$\begin{cases} A_1 \frac{dh_1}{dt} = F_i - \frac{h_1 - h_2}{r_1} \\ A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{r_1} - \left[F_o^{(s)} + K_c (h_2 - h_2^{(s)}) \right] \end{cases}$$



MATLAB implementation

Main

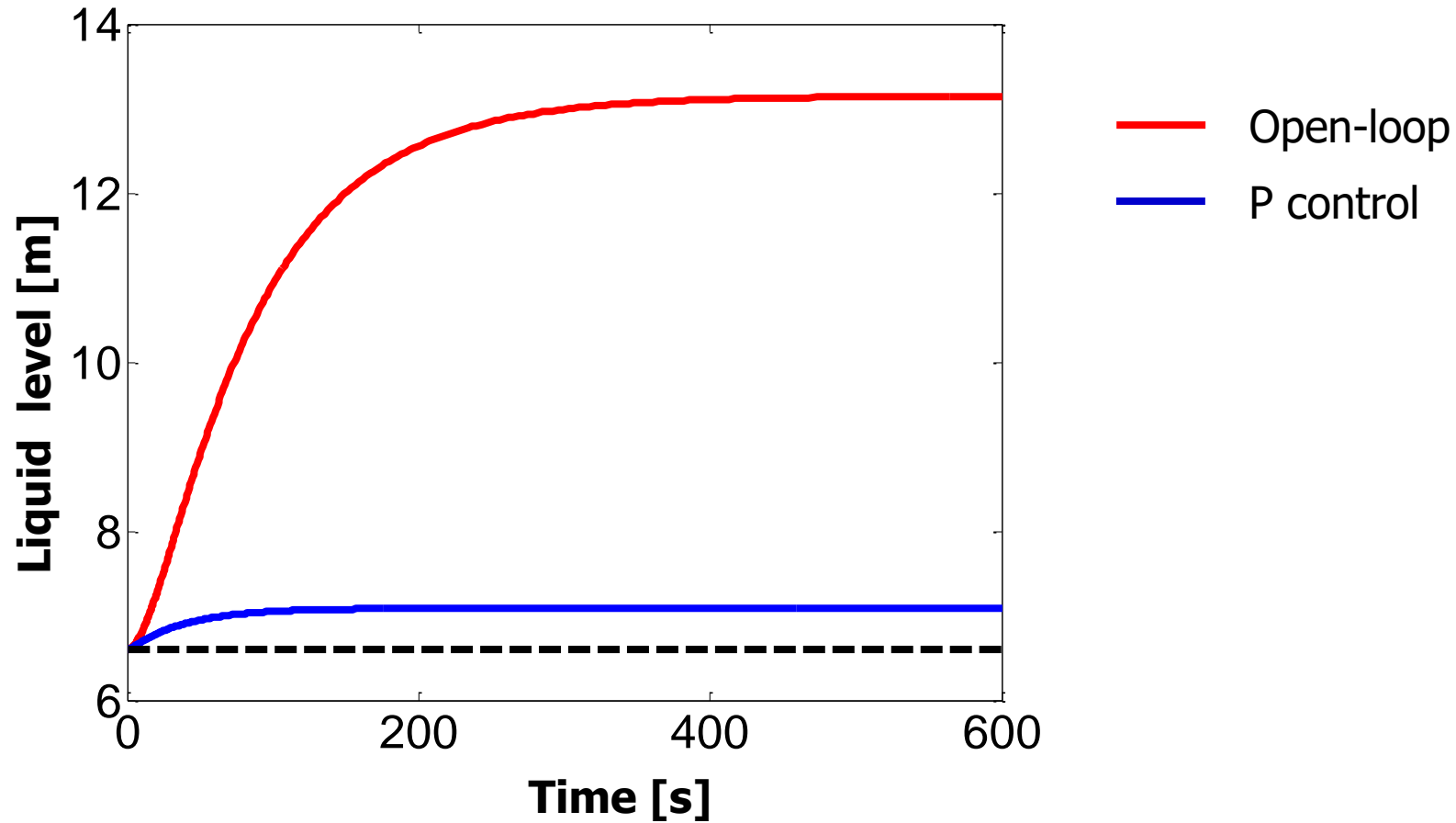
```
Fi_s = 9.4  
y0 = [17.9 6.6]  
tSpan = [0. 600.];  
[t,h] = ode45(@(t,y)Sisdif(t,y,A1,A2,r1,Fi_s),tSpan,y0,options);
```

Sisdif

```
function dy = Sisdif(t,y,A1,A2,r1,Fi_s)  
    Fi = Fi_s*2; %Step  
    h2_s = 6.6; %Set point  
    Fo_s= 1.43*h2_s; %Outlet flow  
  
    dy(1) = (Fi - (h1-h2)/r1) / A1;  
    dy(2) = ((h1-h2)/r1 - (Fo_s + Kc*(h2-h2_s)))/A2;
```



Proportional control



C2- Proportional-Integral controller model

The general form of the model of the proportional-integral control is:

$$c = c_s + K_c (y - y_{SP}) + \frac{K_c}{\tau_I} \int_0^{\bar{t}} (y(t) - y_{SP}) dt$$

Assuming a linear relationship between the valve opening degree and the outlet flowrate, the controller model can be re-written as:

$$F = F_o^{(s)} + K_c (h_2 - h_2^{(s)}) + \frac{K_c}{\tau_I} \int_0^{\bar{t}} (h_2(t) - h_2^{(s)}) dt$$

The model of the system becomes:

$$\begin{cases} A_1 \frac{dh_1}{dt} = F_i - \frac{h_1 - h_2}{r_1} \\ A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{r_1} - \left[F_o^{(s)} + K_c (h_2 - h_2^{(s)}) + \frac{K_c}{\tau_I} \int_0^{\bar{t}} (h_2 - h_2^{(s)}) dt \right] \end{cases}$$

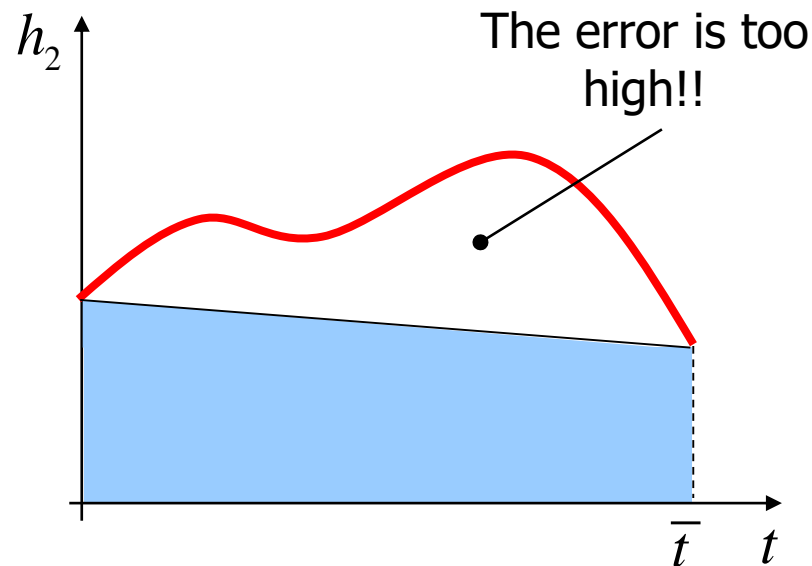


Integral evaluation

PROBLEM: the integral should be evaluated at the current time-step of the ODE system integrator.

The definite integral can be numerically evaluated by means of the trapezoidal rule. For example:

$$I = \int_0^{\bar{t}} (h_2(t) - h_2^{(s)}) dt \approx \frac{\bar{t}}{2} [h_2(\bar{t}) + h_2(0) - 2h_2^{(s)}]$$

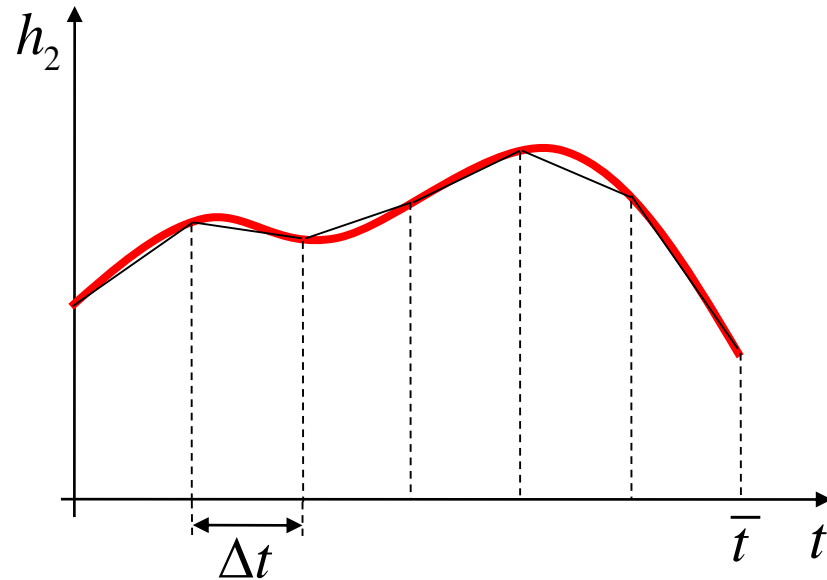


Integral evaluation

It is possible to reduce the numerical error by increasing the number of trapezoids used to fit the integral:

$$I = \int_0^{\bar{t}} \left(h_2(t) - h_2^{(s)} \right) dt \approx \frac{\Delta t}{2} \sum_{n=1}^N \left[h_2((n-1)\Delta t) + h_2(n\Delta t) - 2h_2^{(s)} \right]$$

Number of steps (*a priori* assigned)

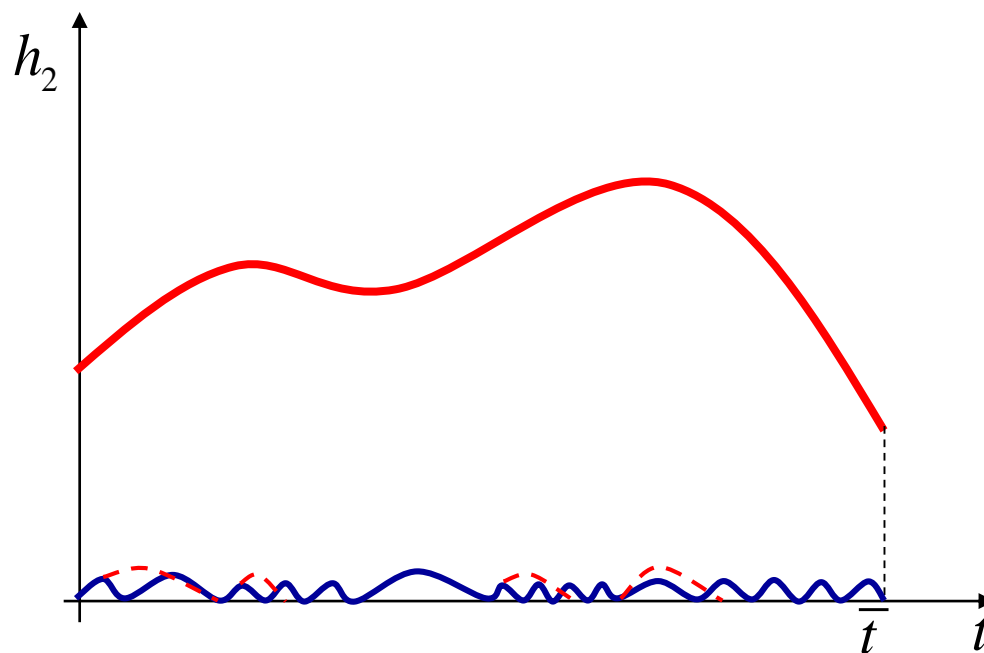


Behaviour of a general integrator

- The integrator calls the function of the differential system is written (Sisdif in the following) at non-equispaced time steps;
- The integrator can re-call the Sisdif at former time steps than the last one;
- Not all the evaluations of the Sisdif contribute to the problem solution. Some of them can be used, for instance, for the evaluation of the Jacobian matrix. Furthermore, if the error overcomes the tolerance, the integration steps fails and it is not used for the solution estimation. In this case, the integration step is reduced.



Behaviour of a general integrator



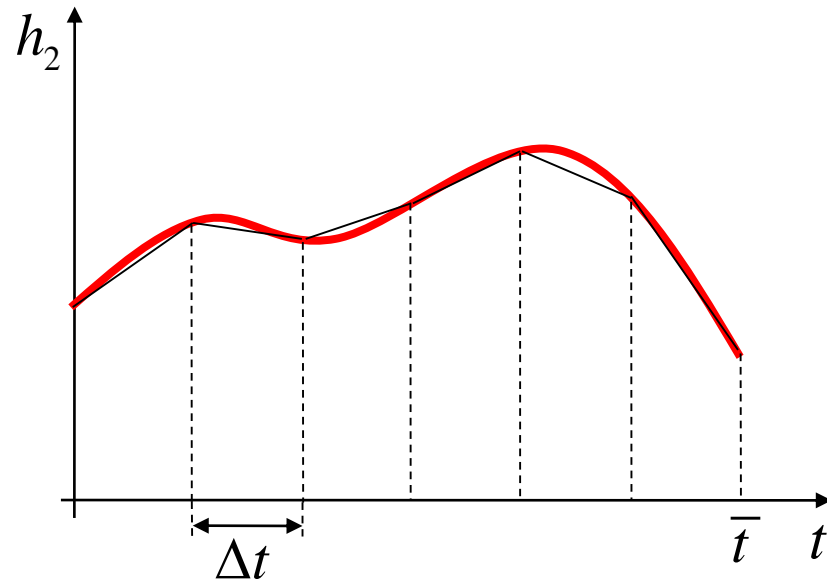
— : successful time step

— : failed step \Rightarrow decrease of the integration step



Solution procedure

1. The integral is solved for equispaced intervals (ex. $\Delta t = 1$ s), i.e. the integration horizon $0 - \bar{t}$ is broken into equispaced sub-intervals:



Solution procedure

2. The differential system is solved in each one of these intervals, evaluating the integral at each step made by the integrator as sum of trapezoids. As reference value to compute the integral, the value of the integral in the first sure former step is used.

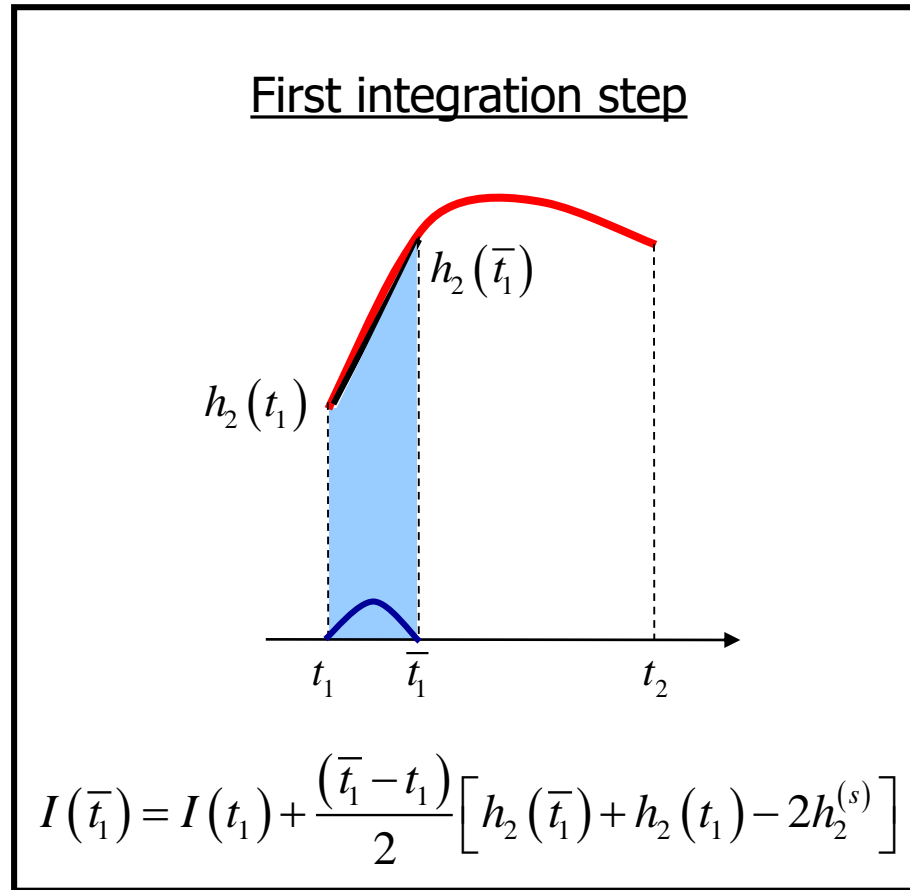
Assume to start from time $t = t_1$ and to integrate up to $t = t_2$. The integral $I(t_1)$ has been formerly computed. The integrator will proceed step-by-step up to reach $t = t_2$. In the intermediate steps, the integral is evaluated as:

$$I = \int_{t_1}^{\bar{t}} \left(h_2(t) - h_2^{(s)} \right) dt = I(t_1) + \frac{(\bar{t} - t_1)}{2} \left[h_2(\bar{t}) + h_2(t_1) - 2h_2^{(s)} \right]$$

with $t_1 < \bar{t} \leq t_2$



Solution procedure



MATLAB implementation

Main

$$I(t + dt) = I(t) + \frac{\Delta t}{2} \left[h_2(t) + h_2(t + \Delta t) - 2h_2^{(s)} \right]$$

Sisdif

$$I(t + d\tau) = I(t) + \frac{\Delta \tau}{2} \left[h_2(t) + h_2(t + \Delta \tau) - 2h_2^{(s)} \right]$$

$$0 < \Delta \tau \leq dt$$



MATLAB implementation (Main)

Variable initialization:

```
global Fi_s r1 A1 A2 h2_s Kc tauI integral tOld fOld

tSpan = [0. 600.];
y0 = [17.9 6.6];           % Initial conditions
A1 = 30., A2 = 50.;       % sqm
r1 = 1.2;                  % s/sqm
Fi_s = 9.4;                % cum/s
h2_s = 6.6;                % Set point
```



MATLAB implementation (Main)

System dynamics evaluation:

```
options = odeset('RelTol',1E-8,'AbsTol',1E-10,'OutputFcn',@Printo);  
[t,y] = ode113(@EXE31P,tSpan,y0,options);  
  
h11 = y(:,1);  
h22 = y(:,2);  
  
variabMeasured = h22(length(h22)); % Output to stationary (take  
the last element of the vector of the tank level 2)  
offSet = abs(vариabMeasured - h2_s); % Offset
```



MATLAB implementation (Sisdif)

```
function dy = EXE31P(t,y)
```

```
    global Fi_s r1 A1 A2 h2_s Kc tauI integral tOld fOld
```

```
        h11 = y(:,1);
```

```
        h22 = y(:,2);
```

```
    . . .
```

```
    Fi = Fi_s*2;
```

```
    h2_s = 6.6;
```

```
    err = h2_s - h2;
```

```
    integralTempo = integral + (fOld + err) * (t - tOld) / 2.;
```

```
    Fout = Fo_s + Kc * err + Kc / tauI * integralTempo;
```

```
    . . .
```

```
    dy(1) = (Fi - (h1-h2)/r1) / A1;
```

```
    dy(2) = ((h1-h2)/r1-Fout)/A2;
```



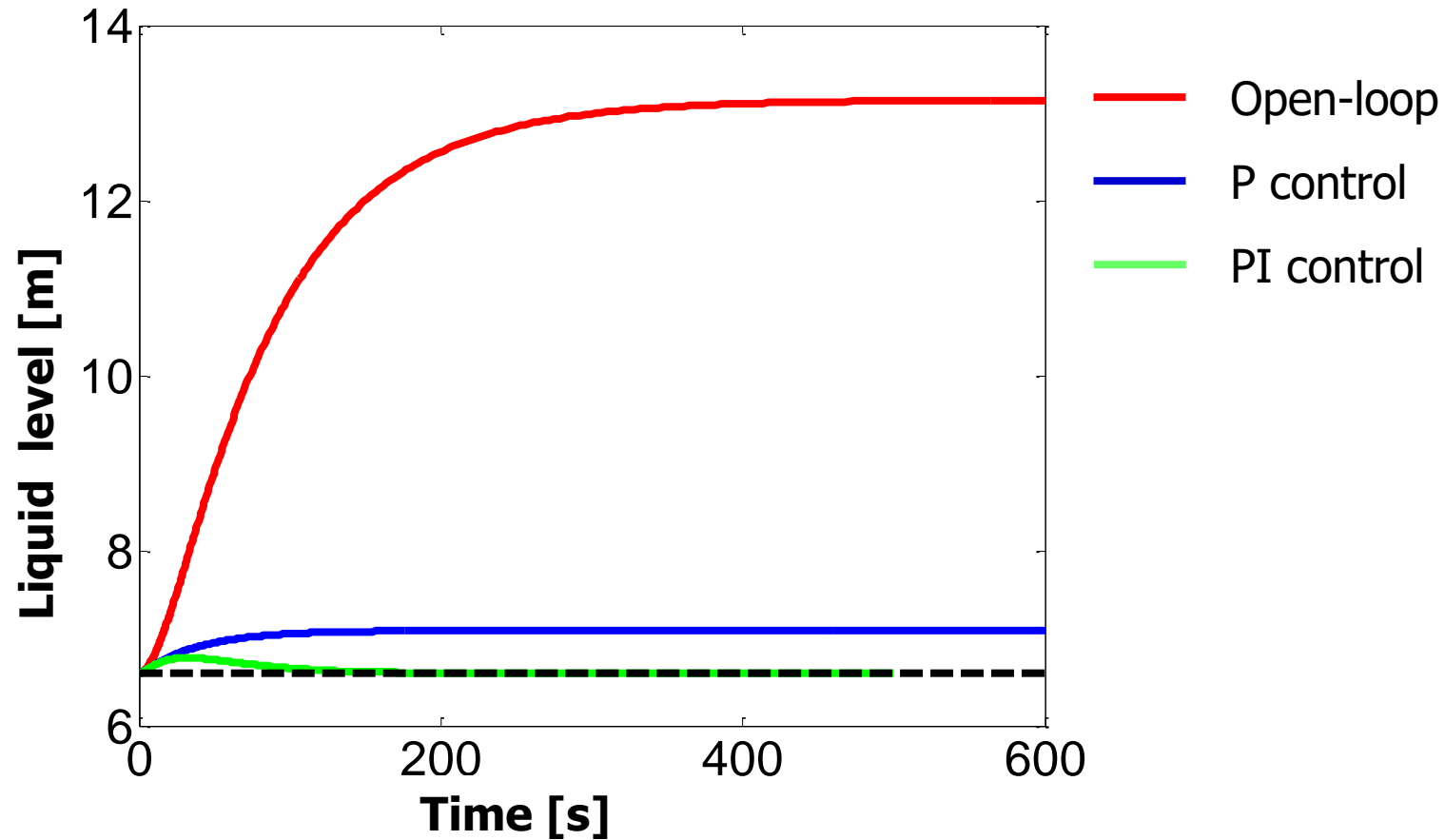
MATLAB implementation (Function)

```
function status = Printo(t,y,flag)
    global integral tOld fOld h2_s
    if strcmp(flag, 'init')
elseif strcmp(flag, 'done')
    else
        h2 = y(2);
        er = h2_s - h2;
        fNew = er;
    tNew = t;
    integral = integral + (fOld + fNew) * (tNew - tOld) / 2.;
    fOld = fNew;
    tOld = tNew;

end
```

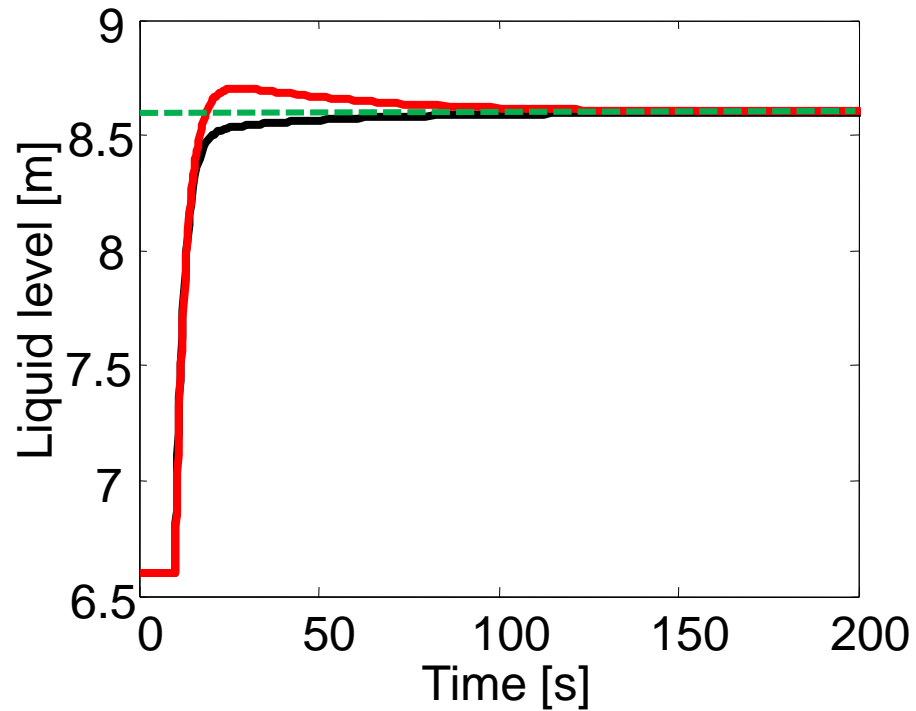


Proportional-Integral controller



Proportional-Integral controller

--- Set point — P control — PI control



Set point change

