SE2

Prof. Davide Manca – Politecnico di Milano

Dynamics and Control of Chemical Processes

Solution to Lab #2

System Dynamics

POLITECNICO
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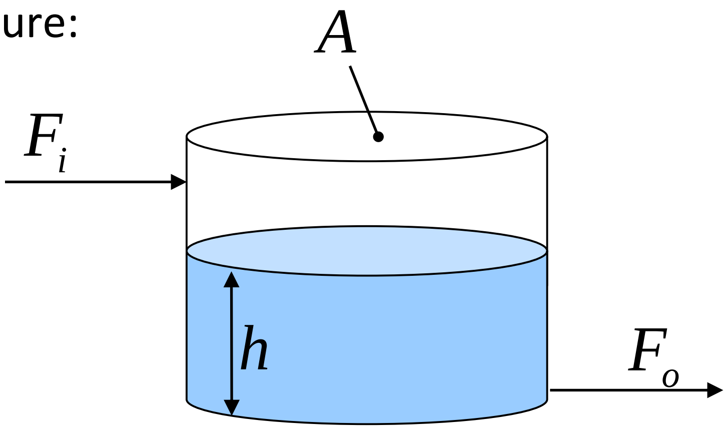
E1 – Dynamics of a tank

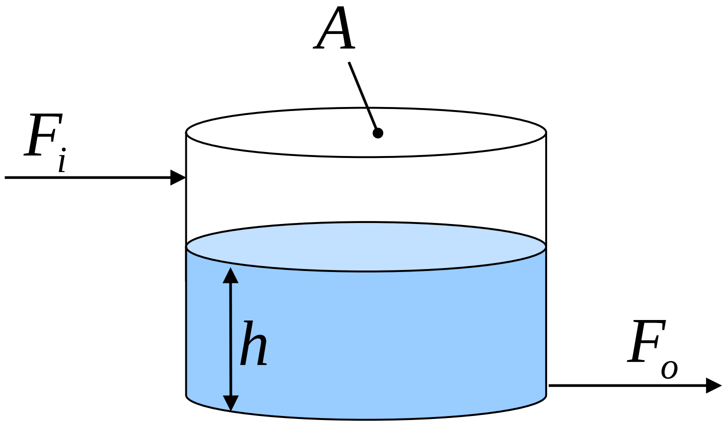
Given the tank in the picture:



1. Evaluate the dynamics of the level of the tank considering a step disturbance on the inlet flowrate, so that it results half of its initial value.
2. Evaluate the level dynamics considering a linear decrease of the inlet flowrate in 30 seconds (ramp variation) up to the half of the initial value.

Remark : the tank is at steady state before the disturbances occur.

Model of the system



$$A\frac{dh}{dt} = F_i - \frac{h}{r}$$

Data:

$$A = 30 \text{ m}^2$$

$$F_i = 7.5 \text{ m}^3/\text{s}$$

$$r = 0.4 \text{ s/m}^2$$

I.C.: $h(0) = h^{(s)}$

Solution procedure question (1)

1. Assessment of the steady state solution:

$$A\frac{dh}{dt}=0=F_i-\frac{h}{r}\qquad\Rightarrow\qquad h^{(s)}=r\cdot F_i=3\text{ m}$$

2. Assignment of the step disturbance on the inlet flowrate:

$$F_i^{(new)}=F_i^{(old)}\big/2=3.75\ \text{m}^3\big/\text{s}$$

3. Evaluation of the system dynamics by integrating the differential equation of the model. The steady state represents the initial conditions.

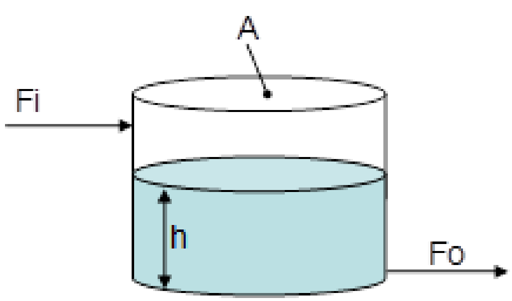
MATLAB implementation

Main

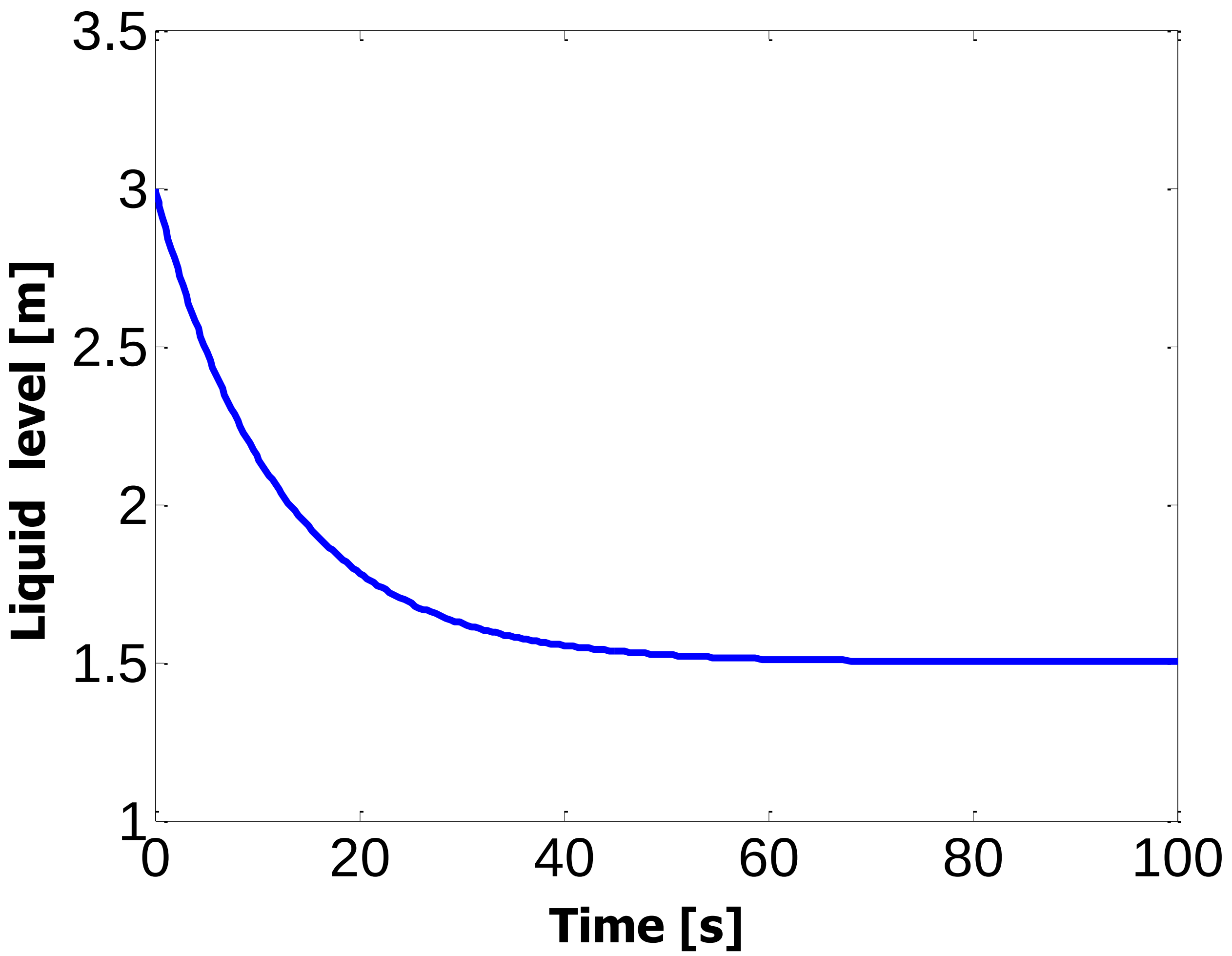
```
Fi0 = 7.5;
h0 = r * Fi0;
tSpan = [0. 100.];

[t,h] = ode45(@(t,y)Sisdif(t,y,A,Fi0,r),tSpan,h0,options);
```

Sisdif

```
function dy = Sisdif(t,y,A,Fi0,r)

h = y(1);
Fi = Fi0/2;

dy(1) = (Fi - h/r)/A
```

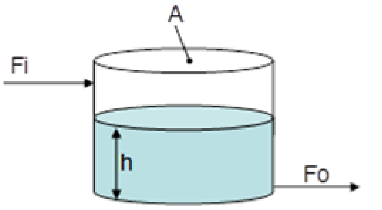


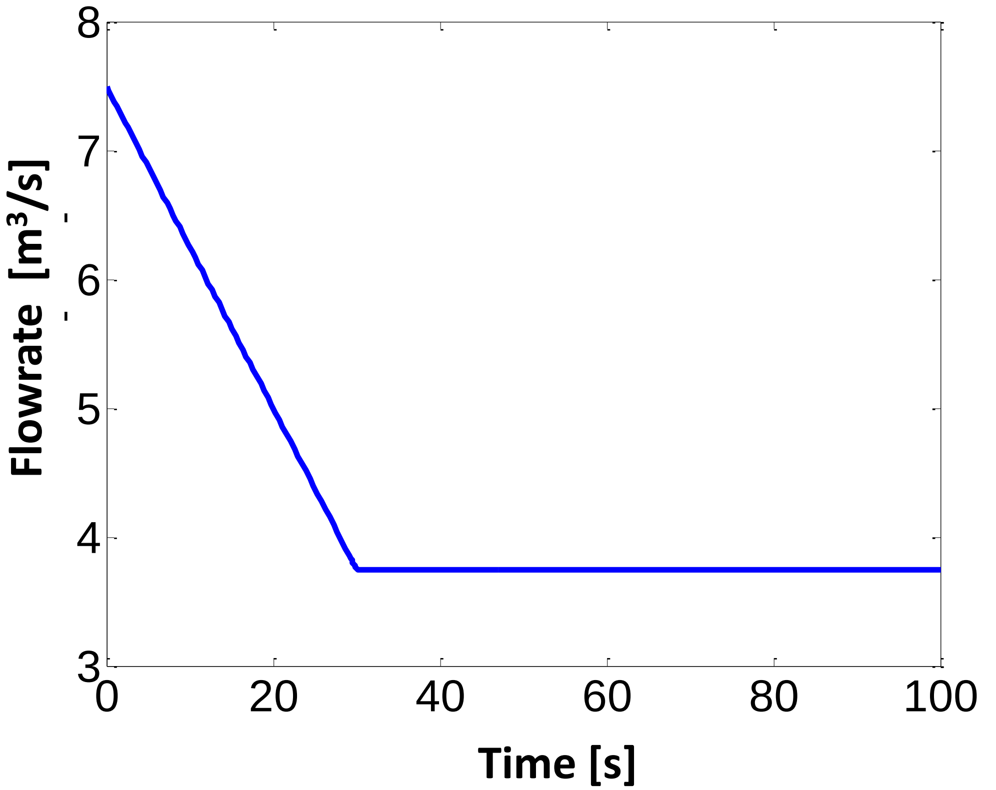
Dynamics in case of a step disturbance



Solution procedure question (2)

- Evaluation of system dynamics by integration of the model differential equation. The inlet flowrate varies with time:





If $t < 30$ s

$$F_i = F_i^{(0)} - \frac{F_i^{(0)}/2}{30} t$$

Else

$$F_i = F_i^{(0)}/2$$

MATLAB implementation

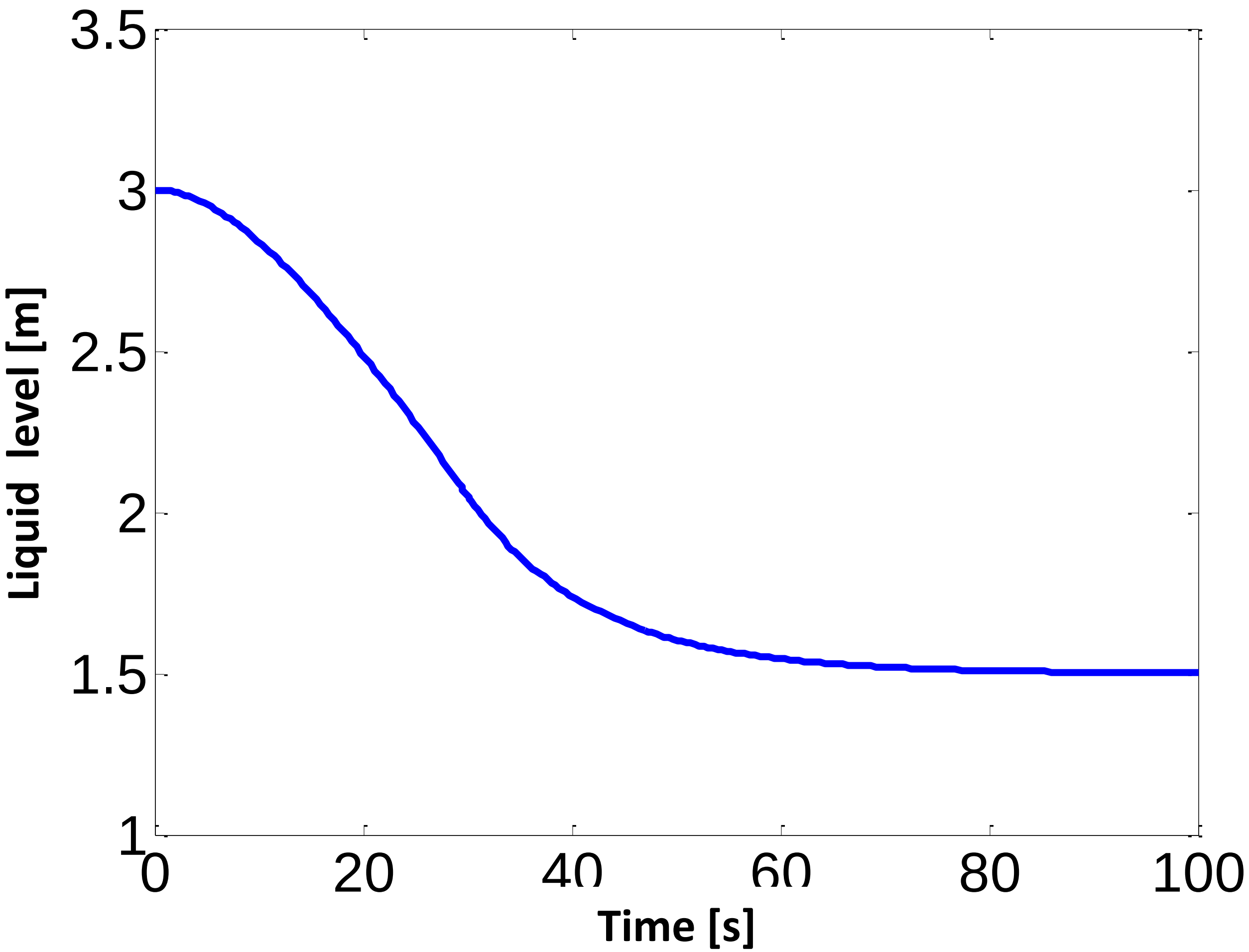
Main

```
Fi0 = 7.5;
h0 = r * Fi0;
[t,h] = ode45(@(t,y)Sisdif(t,y,A,Fi0,r),tSpan,h0,options);
```

Sisdif

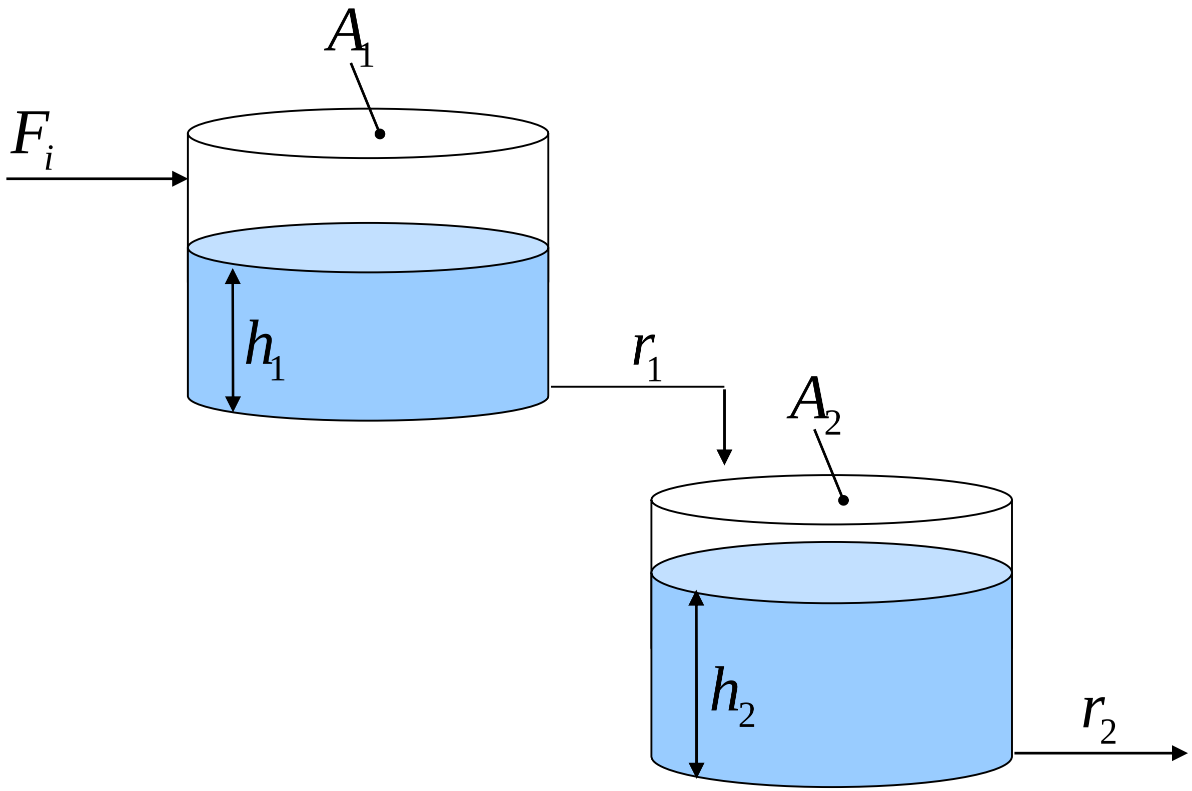
```
function dy = Sisdif(t,y,A,Fi0,r)
   h = y(1);
   if(t < 30)
     Fi = Fi0 - (Fi0/2)/30 * t;
   else
     Fi = Fi0/2;
   end
   dy(1) = (Fi - h/r)/A
```

Dynamics in case of a ramp disturbance



E2a – Dynamics of two non interacting tanks

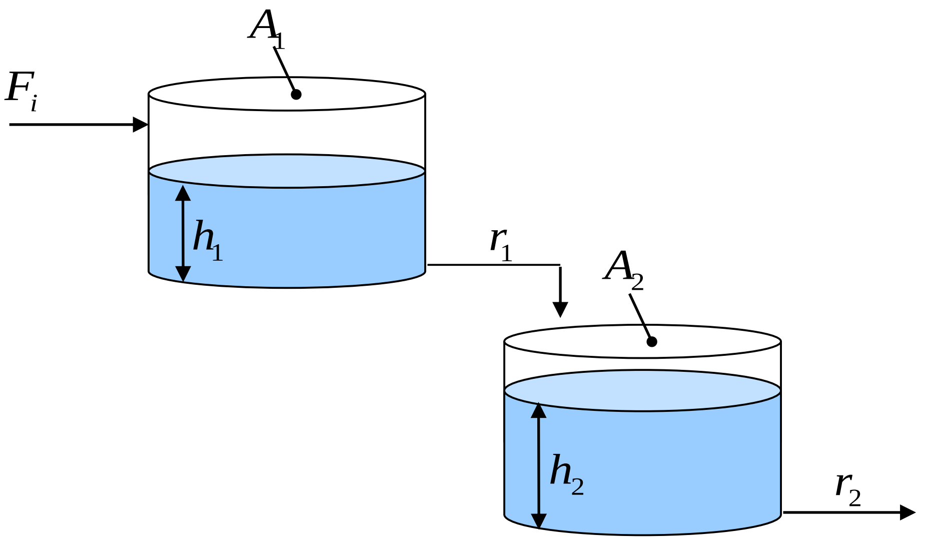
Given the two tanks in the picture:



Evaluate the dynamics of the level of the two tanks in case of a step disturbance on the inlet flowrate, such that halves the initial flowrate.

Remark: the tanks are in steady state before the disturbance occurs.

System model



$$\begin{cases} A_1 \dfrac{dh_1}{dt} = F_i - \dfrac{h_1}{r_1} \\ A_2 \dfrac{dh_2}{dt} = \dfrac{h_1}{r_1} - \dfrac{h_2}{r_2} \end{cases}$$

Data: $F_i = 9.4 \ \mathrm{m^3/s}$

I.C.: Steady state conditions

Tank 1:	Tank 2:
$A_1 = 30 \ \mathrm{m^2}$	$A_2 = 50 \ \mathrm{m^2}$
$r_1 = 1.2 \ \mathrm{s/m^2}$	$r_2 = 0.7 \ \mathrm{s/m^2}$

Solution procedure

1. Assessment of the steady state conditions:

$$\begin{cases} A_1 \dfrac{dh_1}{dt} = 0 = F_i - \dfrac{h_1}{r_1} \\ A_2 \dfrac{dh_2}{dt} = 0 = \dfrac{h_1}{r_1} - \dfrac{h_2}{r_2} \end{cases} \Rightarrow \begin{cases} h_1^{(s)} = r_1 F_i = 11.28 \text{ m} \\ h_2^{(s)} = r_2 F_i = 6.58 \text{ m} \end{cases}$$

2. Assignment of the step disturbance on the inlet flowrate:

$$F_i^{(new)} = F_i^{(old)} / 2 = 4.7 \ \text{m}^3/\text{s}$$

3. Evaluation of the system dynamics by integrating the differential equation of the model. The steady state represents the initial conditions.

MATLAB implementation

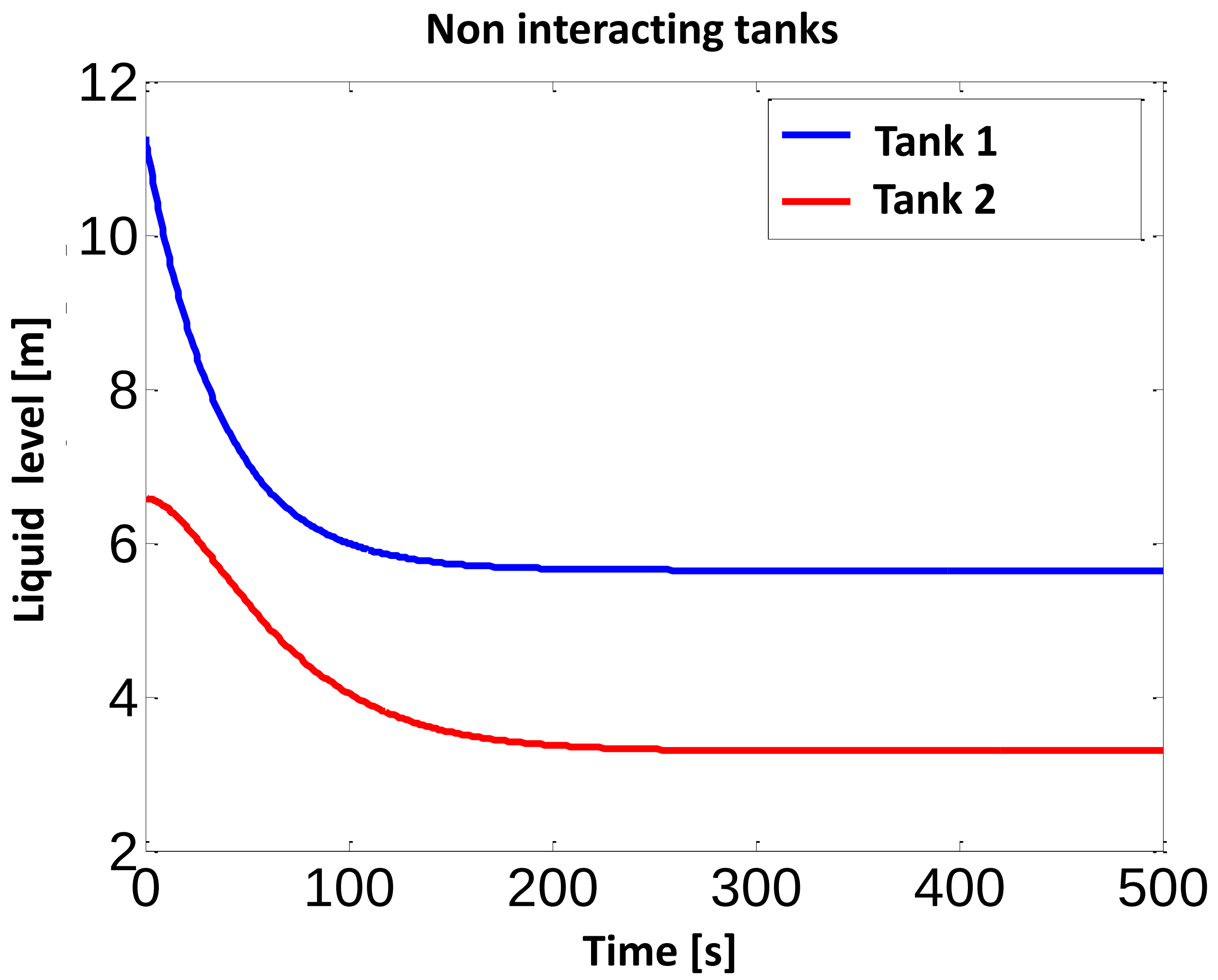
Main

```
Fi0 = 9.4;
h0 = [r1*Fi0 r2*Fi0];
Fi = Fi0/2;
[t,h] = ...
 ode45(@(t,y)Sisdif(t,y,A1,A2,r1,r2,Fi),tSpan,h0,options);
```

Sisdif

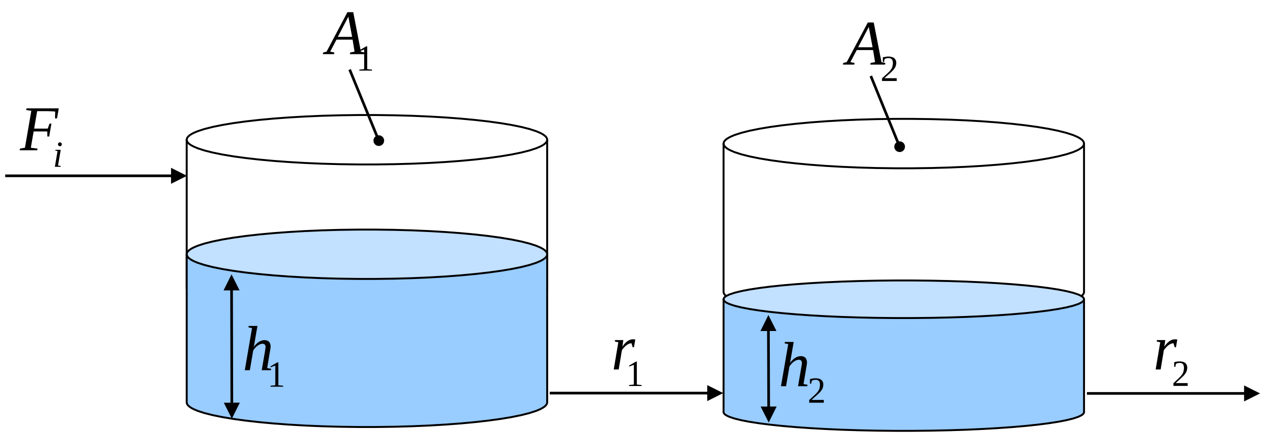
```
function dy = Sisdif(t,y,A1,A2,r1,r2,Fi)
    dy = zeros(2,1);
    h1 = y(1);
    h2 = y(2);
    dy(1) = (Fi - h1/r1)/A1;
    dy(2) = (h1/r1-h2/r2)/A2;
```

Dynamics of two non interacting tanks



E2b - Dynamics of two interacting tanks

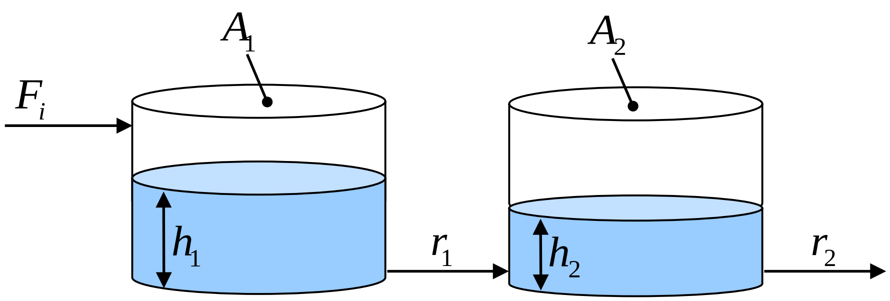
Given the two tanks in the picture:



Evaluate the dynamics of the level of the two tanks in case of a step disturbance on the inlet flowrate, such that it halves the initial flowrate.

Remark: the tanks are in steady state before the disturbance occurs.

System model



$$\begin{cases} A_1 \dfrac{dh_1}{dt} = F_i - \dfrac{h_1 - h_2}{r_1} \\ A_2 \dfrac{dh_2}{dt} = \dfrac{h_1 - h_2}{r_1} - \dfrac{h_2}{r_2} \end{cases}$$

Data: $F_i = 9.4 \ \text{m}^3/\text{s}$

Tank 1:	Tank 2:
$A_1 = 30 \ \text{m}^2$	$A_2 = 50 \ \text{m}^2$
$r_1 = 1.2 \ \text{s/m}^2$	$r_2 = 0.7 \ \text{s/m}^2$

I.C.: Steady state conditions

Solution procedure

1. Evaluation of steady state conditions:

$$\begin{cases} A_1 \dfrac{dh_1}{dt} = 0 = F_i - \dfrac{h_1 - h_2}{r_1} \\ A_2 \dfrac{dh_2}{dt} = 0 = \dfrac{h_1 - h_2}{r_1} - \dfrac{h_2}{r_2} \end{cases} \Rightarrow \begin{cases} h_1^{(s)} = \left(r_1 + r_2\right) F_i = 17.86 \text{ m} \\ h_2^{(s)} = r_2 F_i = 6.58 \text{ m} \end{cases}$$

2. Assignment of the step disturbance on the inlet flowrate:

$$F_i^{(new)} = F_i^{(old)} / 2 = 4.7 \ \ \text{m}^3/\text{s}$$

3. Evaluation of the system dynamics by integration of the differential equation of the model. The steady state represents the initial conditions.

MATLAB implementation

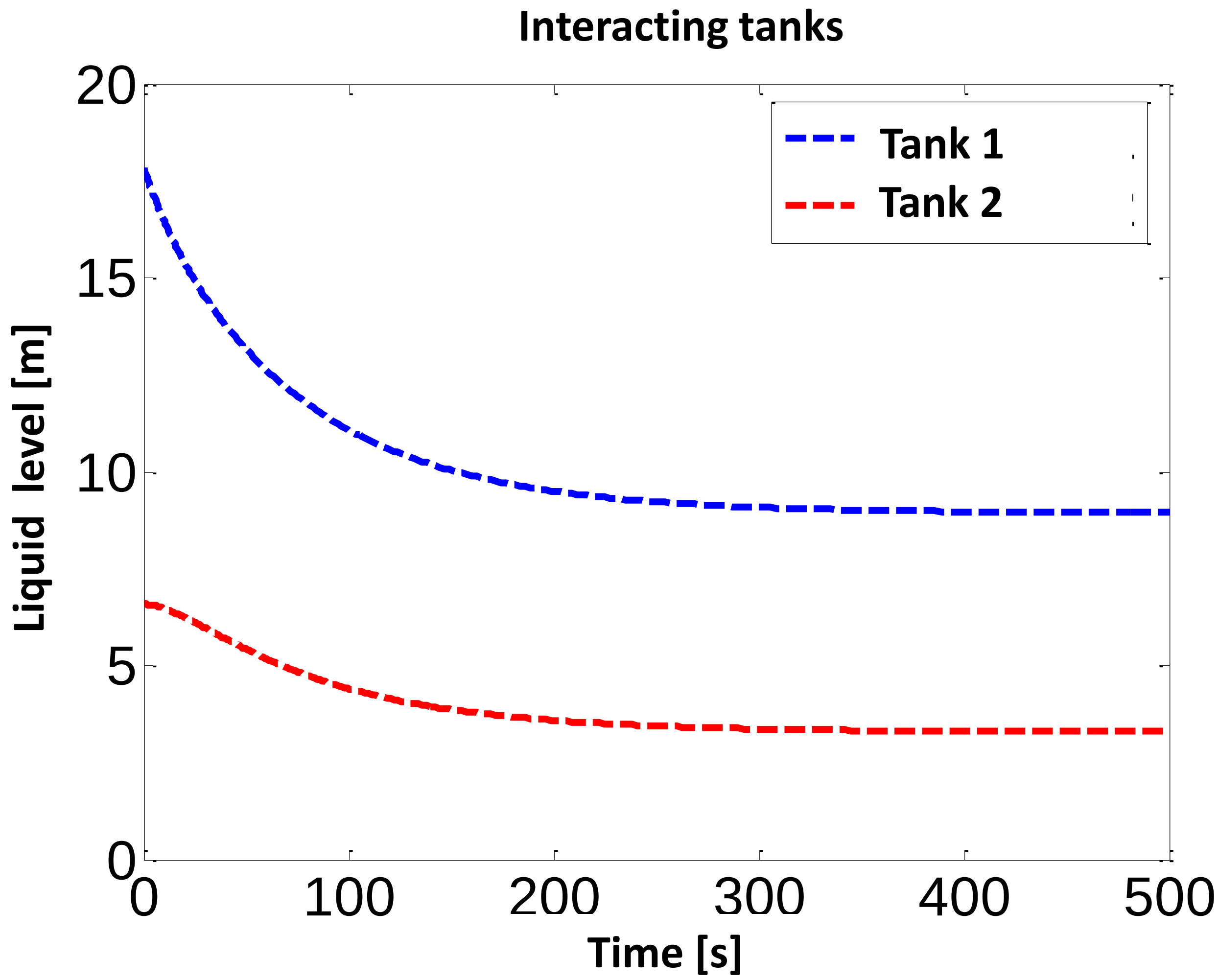
Main

```
Fi0 = 9.4;
h0 = [(r1+r2)*Fi0 r2*Fi0];
Fi = Fi0/2;
[t,h] = ode45(@(t,y)Sisdif(t,y,A1,A2,r1,r2,Fi),tSpan,h0,options);
```

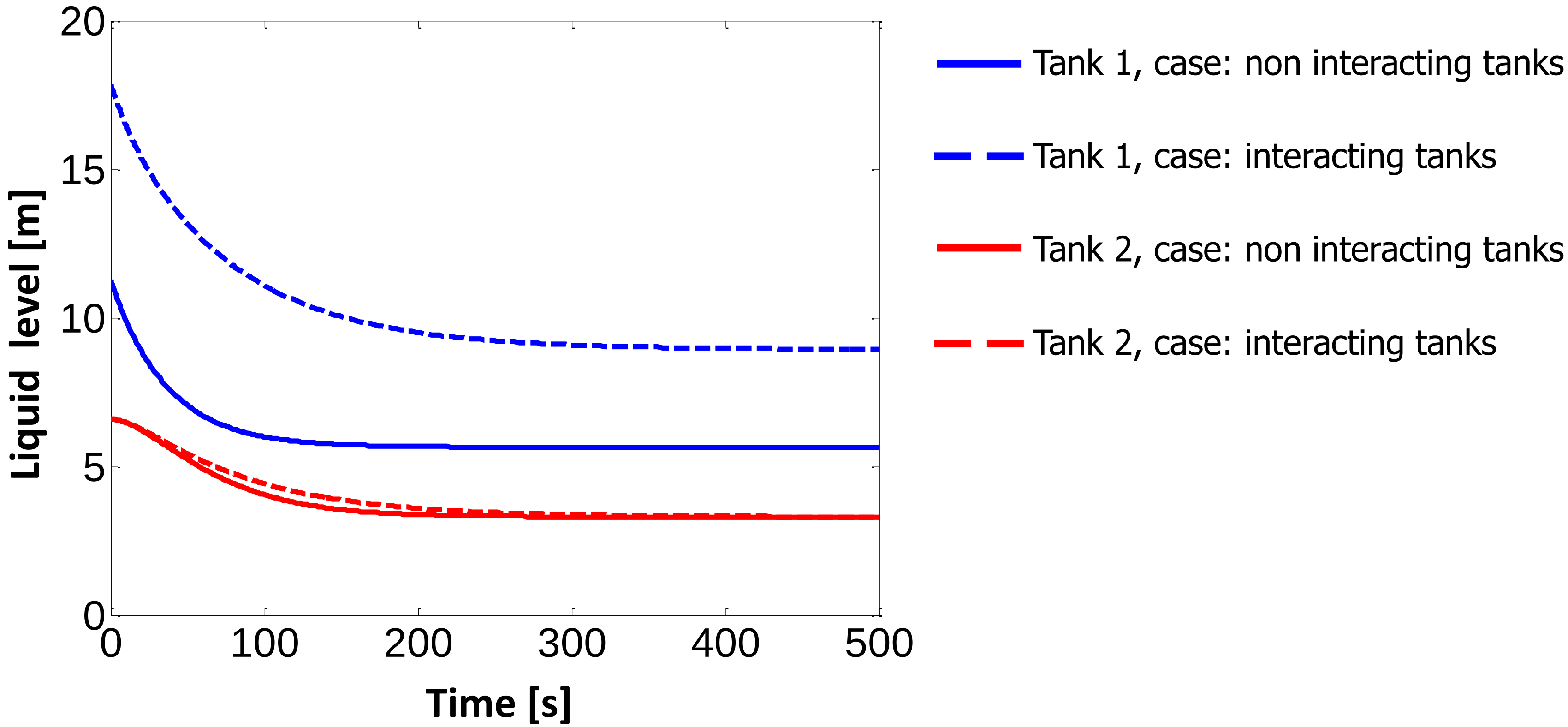
Sisdif

```
function dy = Sisdif(t,y,A1,A2,r1,r2,Fi)
    dy = zeros(2,1);
    h1 = y(1);
    h2 = y(2);
    dy(1) = (Fi - (h1-h2)/r1)/A1;
    dy(2) = ((h1-h2)/r1-h2/r2)/A2;
```

Dynamics of interacting tanks



Comparison



20
15
10
5
0
0
100
200
300
400
500
Liquid level [m]
Time [s]
Tank 1, case: non interacting tanks
Tank 1, case: interacting tanks
Tank 2, case: non interacting tanks
Tank 2, case: interacting tanks