Dynamics and control of incineration processes

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Lesson 2 of "Dynamics and Control of Chemical Processes" – Master Degree in Chemical Engineering

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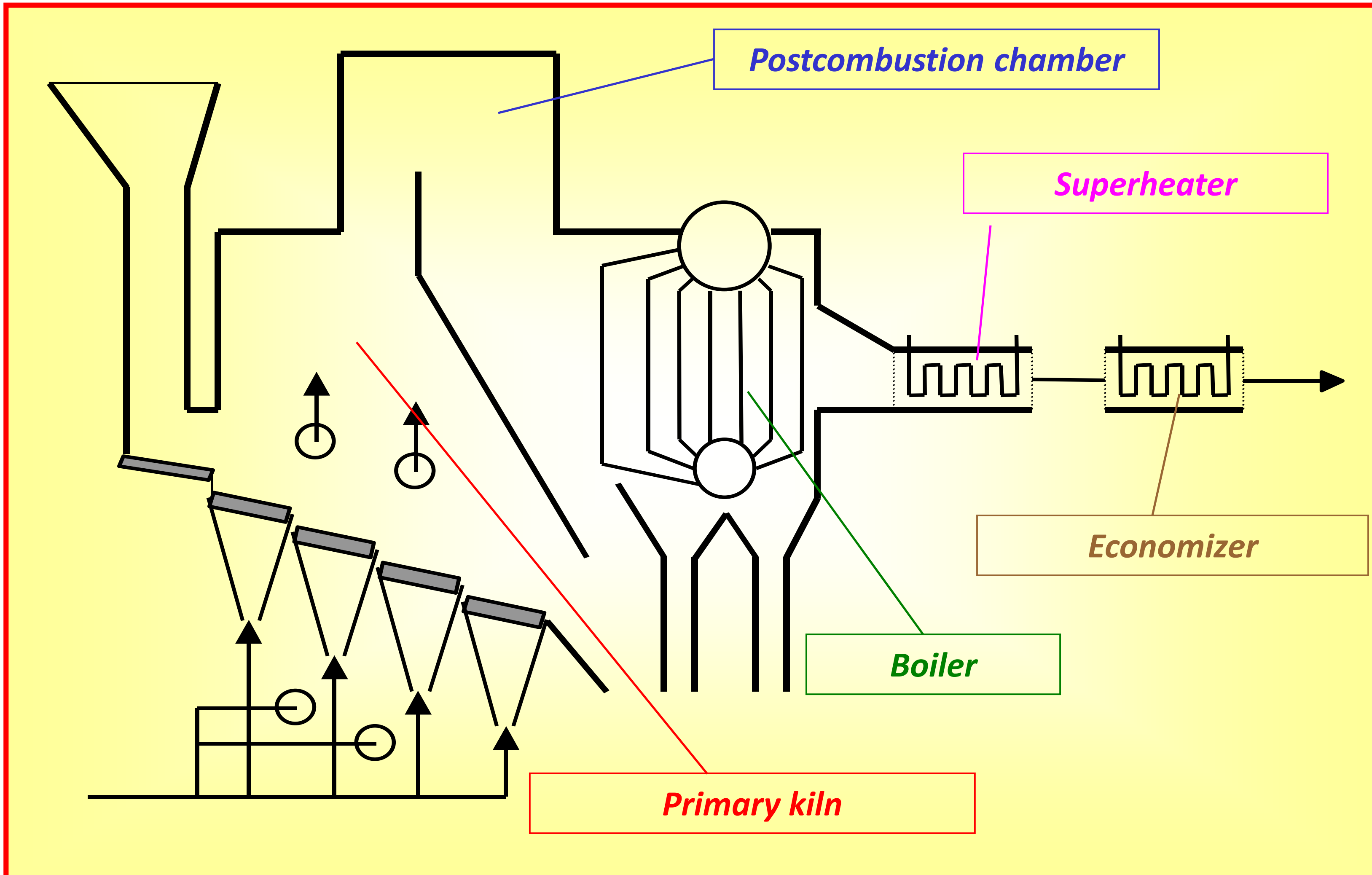
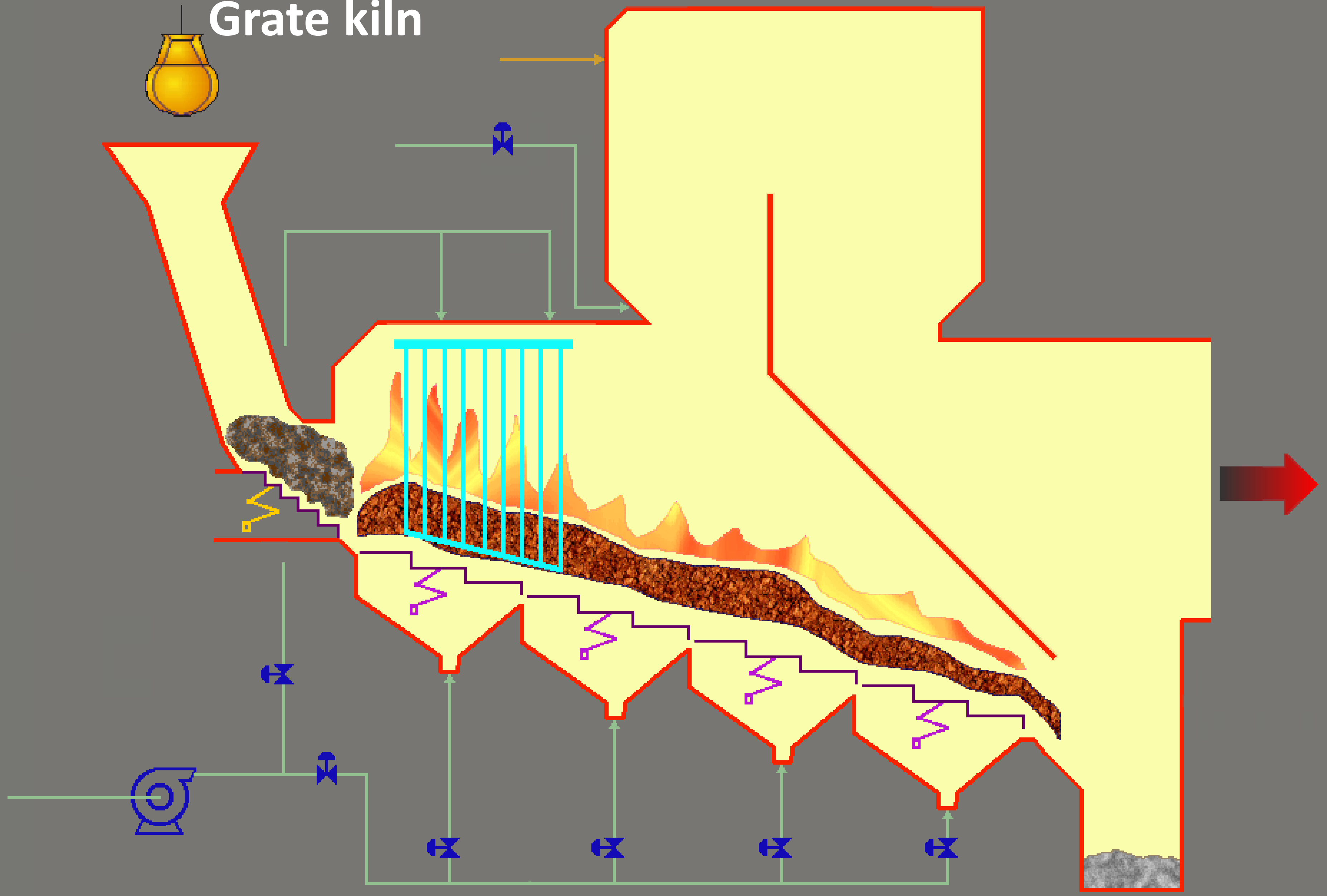
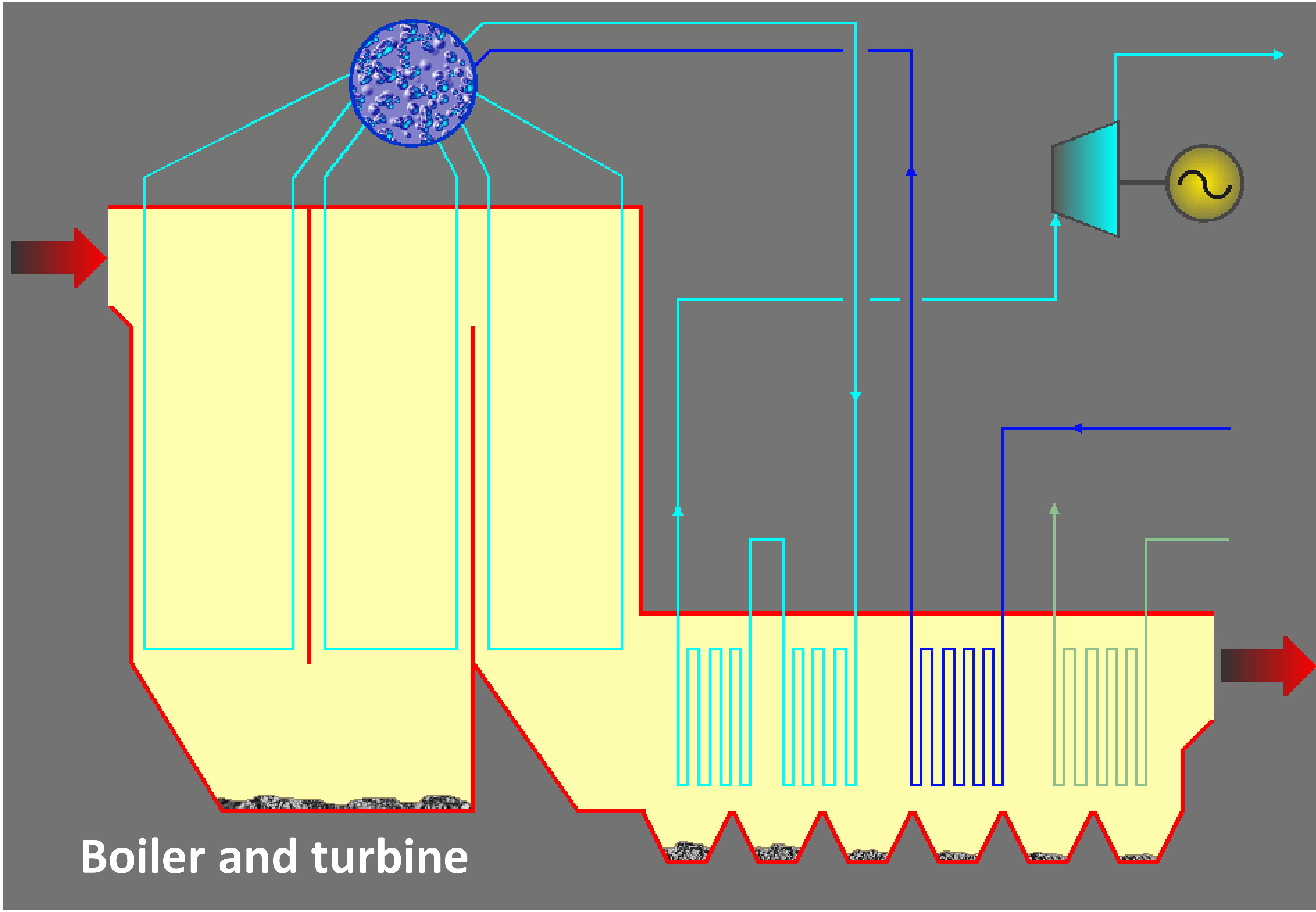
Hot section of an incineration plant



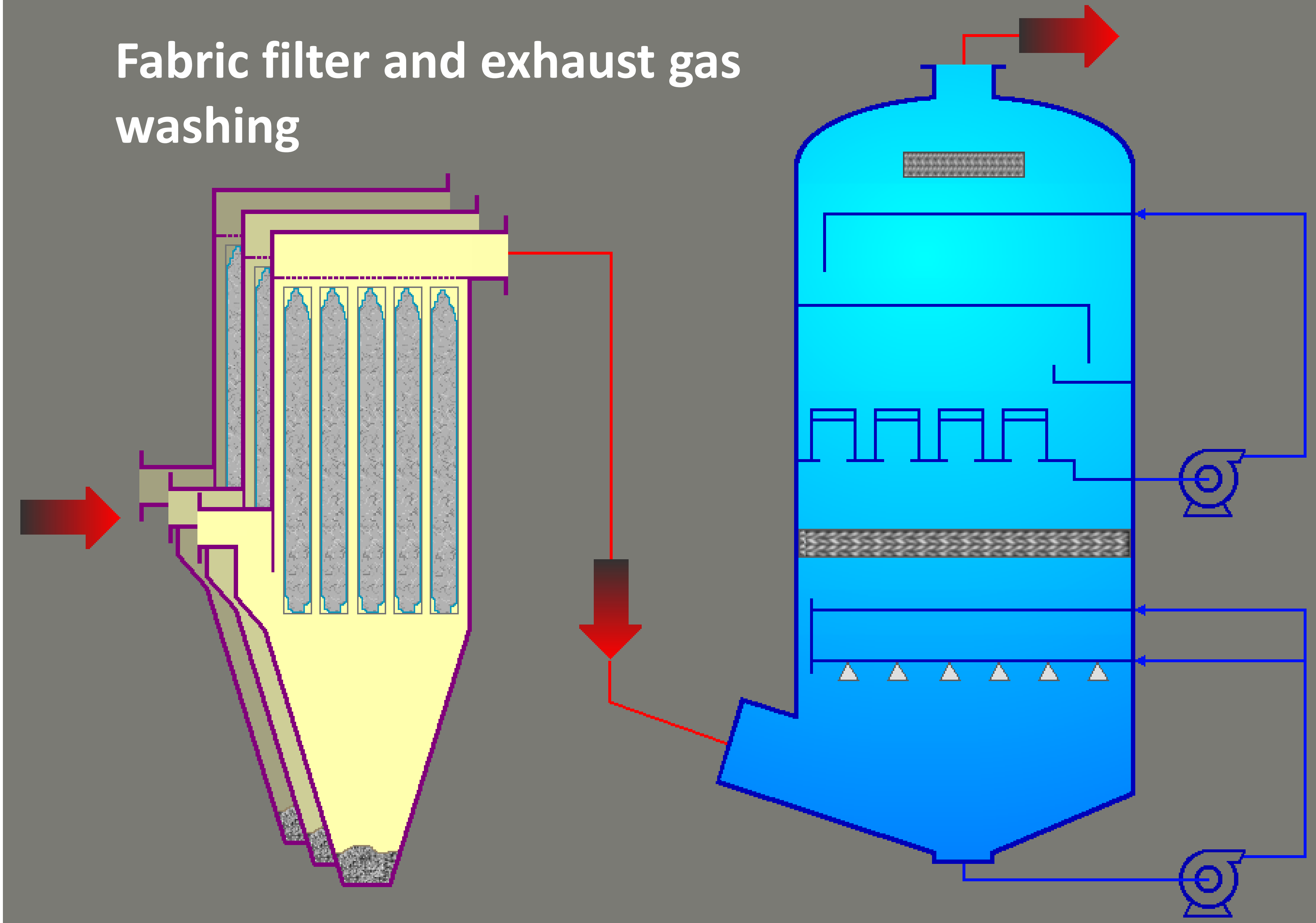
Grate kiln



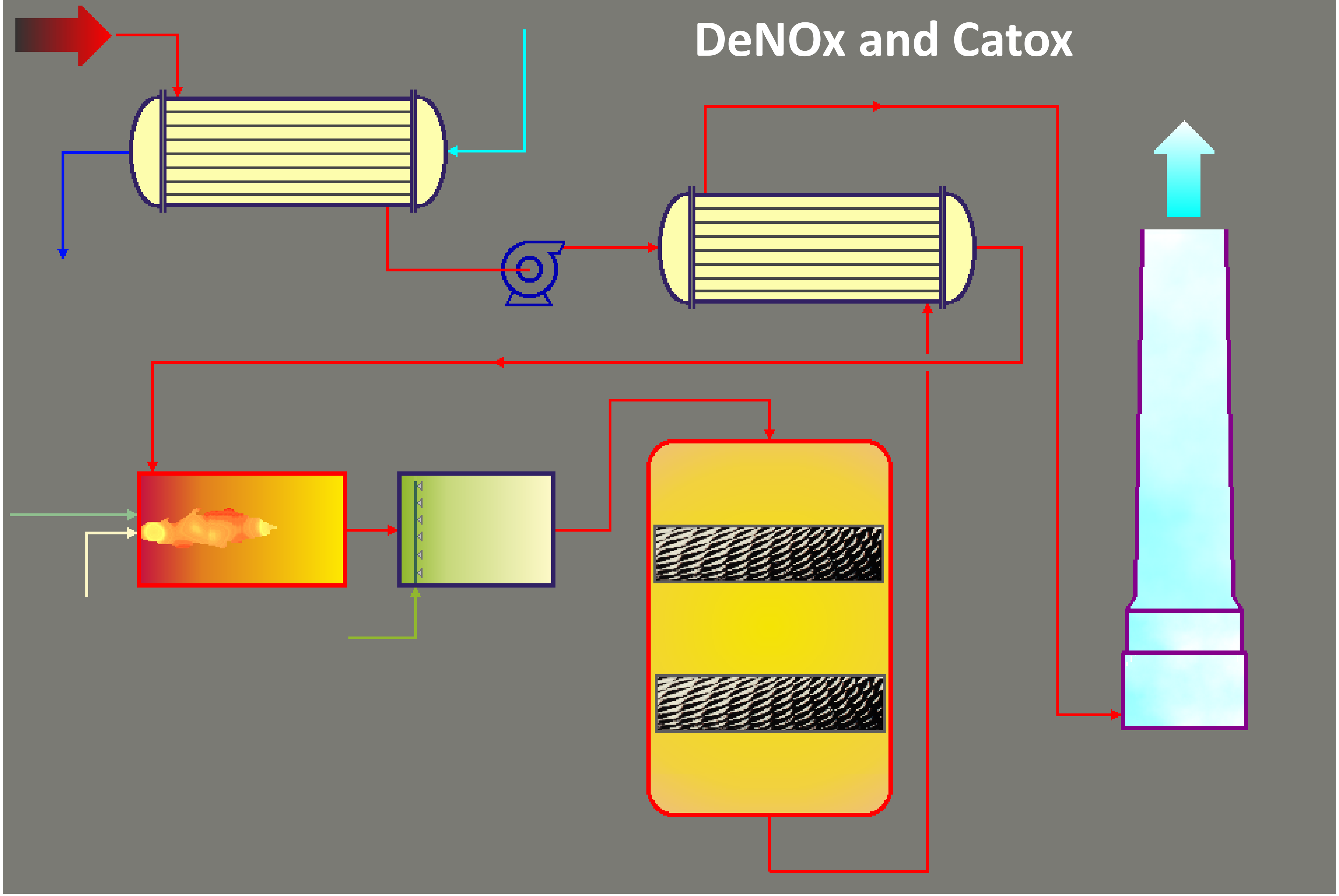


Boiler and turbine

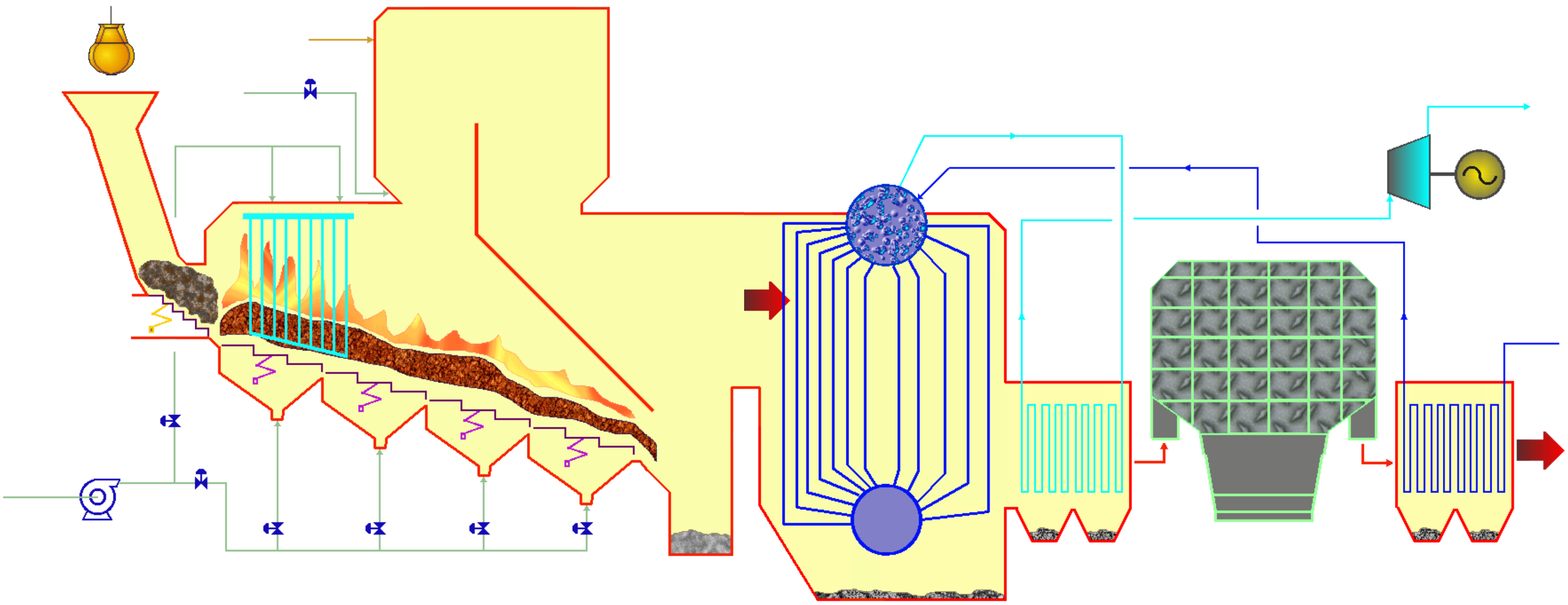
Fabric filter and exhaust gas washing



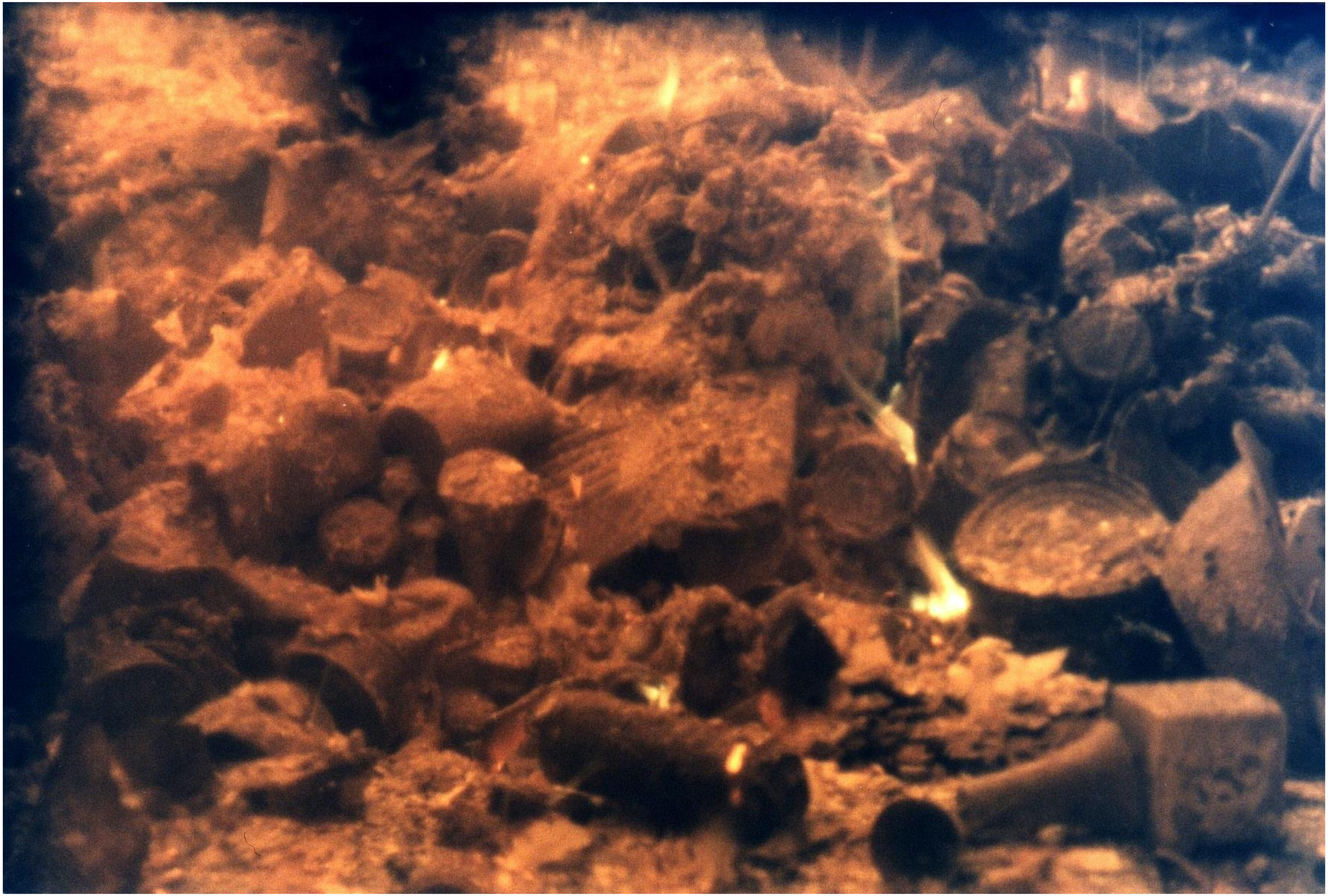
DeNOx and Catox



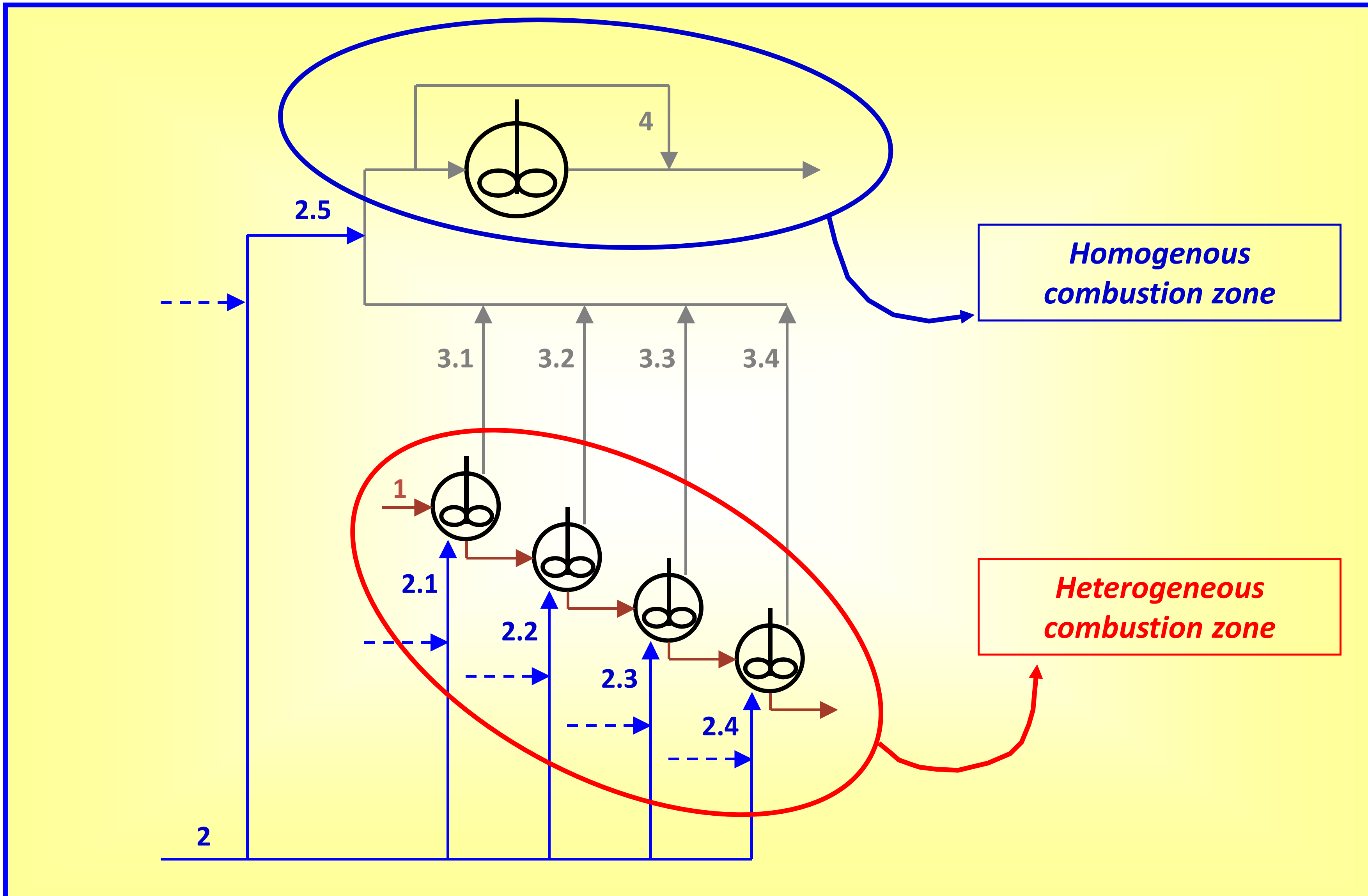
Layout of an incineration plant



Primary combustion chamber



Primary kiln – Modeling scheme



Primary kiln – Material bal. on the grates

Solid Phase

$$\frac{dM_{RIF,i}}{dt} = F^{IN}_{RIF,i} - F^{OUT}_{RIF,i} - R_{RIF,i}$$

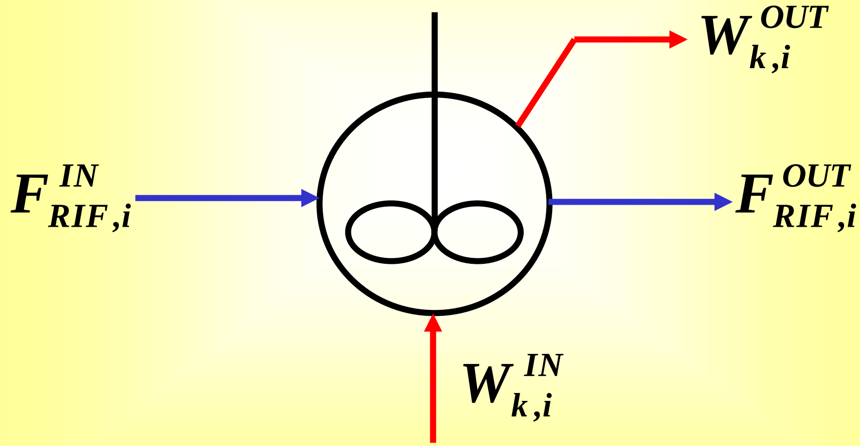
$$i = 1, NG$$

Gas Phase

$$W^{OUT}_{k,i} + \frac{R_{RIF,i}}{PM_{rif}} \cdot x_k - W^{OUT}_{k,i} = 0$$

$$k = 1, NC \quad i = 1, NG$$

i-th grate



Kinetics

Combustion reaction in the solid phase

$$C_nH_mS_pO_qCl_kN_y \xrightarrow{\mu_{O_2,i}} nCO + kHCl + pSO_2 + x_iNO + \frac{y - x_i}{2}\cdot N_2 + \frac{m-k}{2}\cdot H_2O$$

$$x_i = \psi_{NO,i}\cdot y$$

$$\psi_{NO,i} = \frac{2}{\dfrac{1}{\psi_{NO,i}} - \dfrac{2500\cdot W_{G\,NO,i}^{100\%}}{T\cdot exp(-3150/T)\cdot W_{G\,O_2,i}^{out}}} - 1$$

(Bowman, 1975)

Combution reactions in the gas phase

$$CO + \frac{1}{2}\cdot O_2 \rightarrow CO_2 \qquad \frac{1}{2}\cdot N_2 + \frac{1}{2}\cdot O_2 \rightarrow NO$$

Primary kiln – Combustion kinetics

Kinetic determining step: O_2 diffusion

$$R_{waste,i} = \frac{k_{x,i} \cdot x_{O_2,i}}{\mu_{O_2,i}} \cdot A^*_{sc,i} \cdot PM_{waste}$$

$\mu_{O_2,i}$ see L2–18 eq. (8)

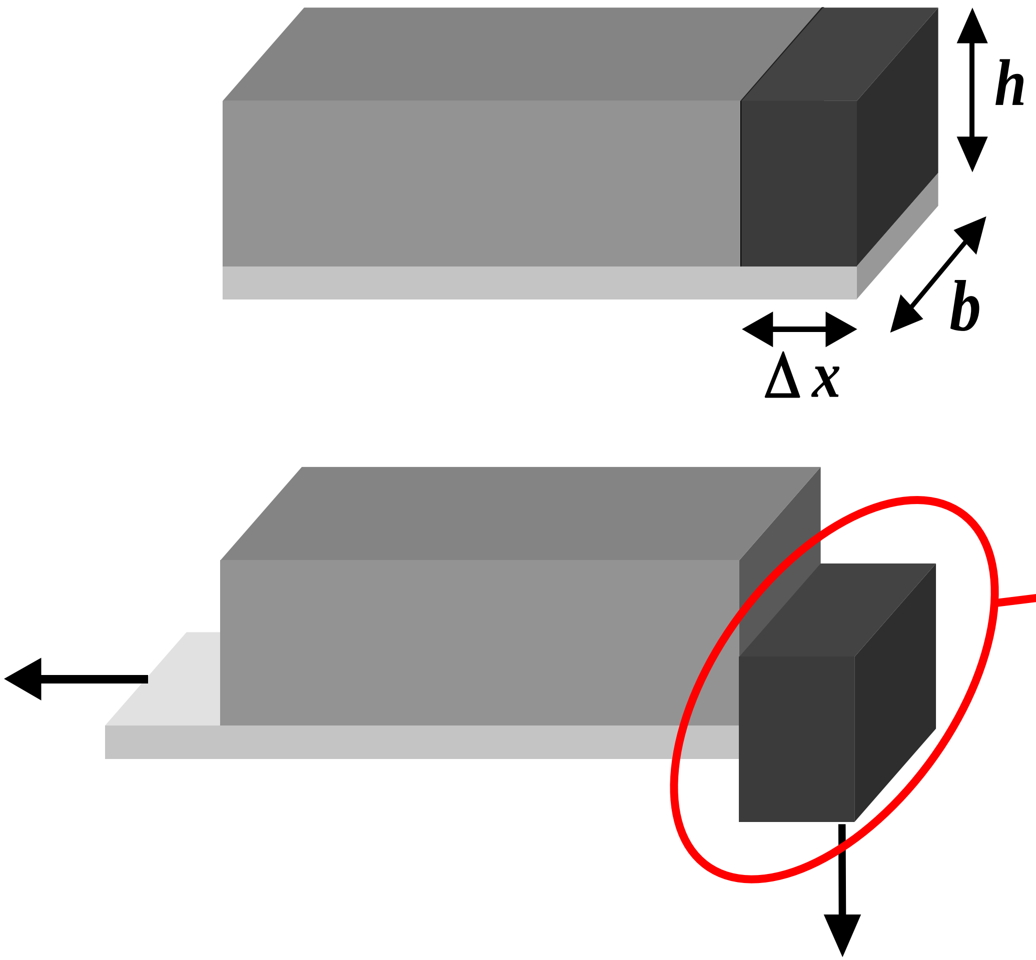
$$\begin{cases} A_{sc,i} = f(d_{p,i}, \varepsilon_i, M_{waste,i}, M_{inerts,i}) \\ \boldsymbol{k_{x,i} = f(d_{p,i}, \varepsilon_i, W_{aria,i}, x_{k,i}, T)} \end{cases}$$

Solid granular bed

$$\begin{cases} \boldsymbol{A^*_{sc,i}} = \Gamma_{\boldsymbol{i}} \cdot \boldsymbol{A_{sc,i}} \\ \Gamma_{\boldsymbol{i}} = 1 + \delta \cdot \left(1 - \exp\left(-\frac{\boldsymbol{N_{CG,i}}}{\lambda}\right)\right) \end{cases}$$

Corrective factor with adaptive parameters δ, λ

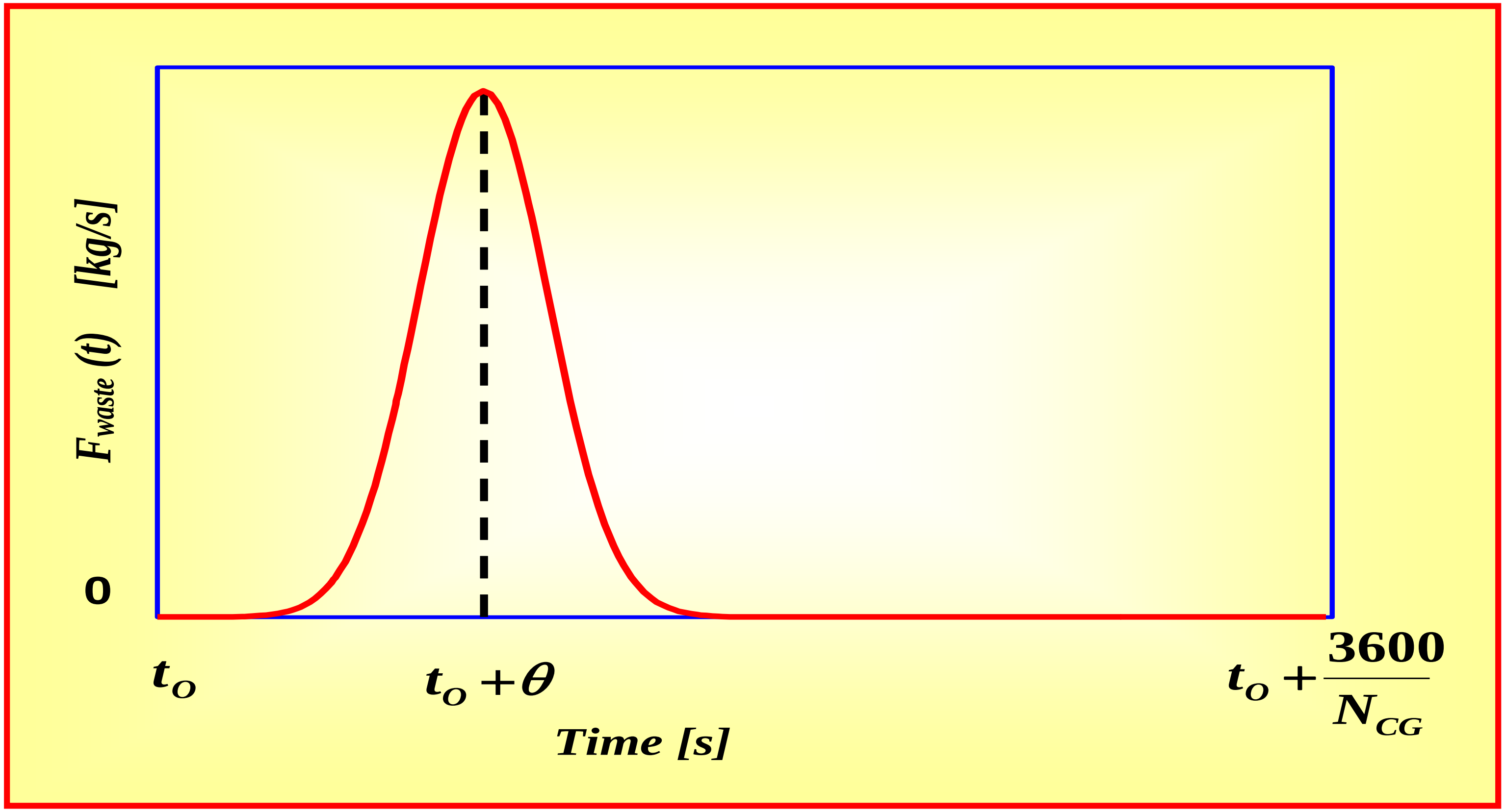
Grate schematization



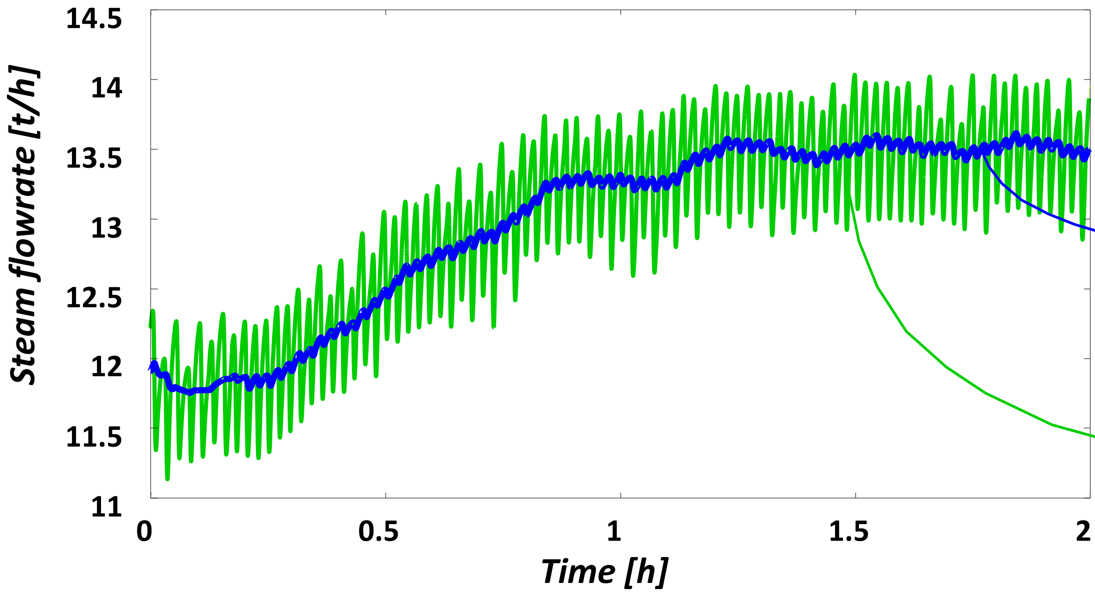
$$\Delta m = \rho_{waste} \cdot \Delta x \cdot b \cdot h$$

Waste pulse schematization

$$F_{waste}(t) = \Delta m \cdot \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma^2}} \cdot \exp\left(-\frac{(t-(t_0+\theta))^2}{2 \cdot \sigma^2}\right)$$



Moving average

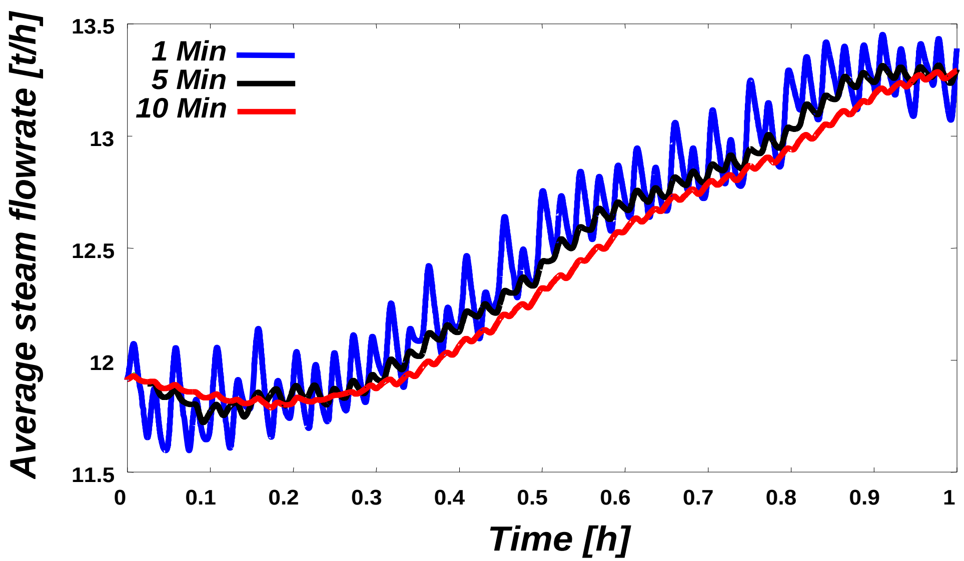


Average value

$$\bar{\mathbf{y}} = \frac{\sum_{k=1}^{NT} \mathbf{y}_k}{NT}$$

$$NT = \frac{t_{MEDIA}}{T_S}$$

Valore istantaneo



$t_{average}$

- ***High* : delay time**
- ***Low* : high oscillations**

Model equations

Manca D., M. Rovaglio, Ind. Eng. Chem. Res. 2005, *44*, 3159-3177

$$\frac{dM_{W,i}}{dt} = F_{W,i}^{in} - F_{W,i}^{out} - R_{W,i} \quad i = 1,\ldots,NG \qquad (1)$$

$$\frac{dM_{I,i}}{dt} = F_{I,i}^{in} - F_{I,i}^{out} \quad i = 1,\ldots,NG \qquad (2)$$

$$F_{W,i}^{in} = F_{W,i-1}^{out}, F_{I,i}^{in} = F_{I,i-1}^{out} \quad i = 2,\ldots,NG \qquad (3)$$

$$F_{W,1}^{in} = F_W(1 - \omega_{I,0} - \omega_{M,0}) \qquad (4)$$

$$F_{I,1}^{in} = F_W\omega_{I,0} \qquad (5)$$

$$C_nH_mS_pO_qCl_kN_y \xrightarrow{\mu_{O_2,i}} nCO + kHCl + pSO_2 + x_iNO + \frac{y - x_1}{2}N_2 + \frac{m-k}{2}H_2O \qquad (6)$$

$$R_{W,i} = \frac{k_{x,i}x_{O_2,i}}{\mu_{O_2,i}} \qquad (7)$$

$$\mu_{O_2,i} = \frac{1}{2}\left(n + 2p + x_i + \frac{m-k}{2} - q\right) \qquad (8)$$

$$k_{x,i} = Sh_i \mathscr{D}_{O_2,i}^{mix} a_i c_{tot,i} \qquad (9)$$

$$a_i = \frac{6(1 - \epsilon_i)}{d_{p,i}} \qquad (10)$$

$$Sh_i = j_{D,i} Re_i Sc_i^{1/3} \qquad (11)$$

$$\begin{cases} Re_i = \dfrac{\rho_A v_{A,i}}{\mu_A a_i} \\ Sc_i = \dfrac{\mu_A}{\rho_A \mathscr{D}_{O_2,i}^{mix}} \end{cases} \qquad (12)$$

$$A_{E,i} = \Gamma_i \frac{a_i}{(1 - \epsilon_i)} f_{W,i} \frac{M_{W,i} + M_{I,i}}{\rho_W} \qquad (13)$$

$$\Gamma_i = 1 + \delta(1 - e^{-N_{GS,i}/\lambda}) \qquad (14)$$

Model equations

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$$\frac{\Delta P_i}{s_{B,i}} = \frac{150\mu_A v_{A,i}(1-\epsilon_i)^2}{d_{p,i}{}^2\epsilon_i{}^3} + \frac{1.75\rho_A v_{A,i}{}^2(1-\epsilon_i)}{d_{p,i}\epsilon_i{}^3} \qquad (15)$$

$$s_{B,i} = \frac{M_{W,i} + M_{I,i}}{\rho_W l_G w_G} \qquad (16)$$

$$\begin{cases}
(W_{A,G,i} + W_{A,L,G,i})x_{O_2,A} - \dfrac{R_{W,i}\mu_{O_2}}{MW_W} - W^{out}_{O_2,G,i} = 0 \\
(W_{A,G,i} + W_{A,L,G,i})x_{N_2,A} - \dfrac{R_{W,i}}{MW_W}\dfrac{x_{N,W}}{2}(1-\Psi_{NO,i}) - \\
\qquad W^{out}_{N_2,G,i} = 0 \\
\dfrac{R_{W,i}}{MW_W}x_{C,W} - W^{out}_{CO,G,i} = 0 \\
\dfrac{R_{W,i}}{MW_W}x_{S,W} - W^{out}_{SO_2,G,i} = 0 \\
\dfrac{R_{W,i}}{MW_W}x_{Cl,W} - W^{out}_{HCl,G,i} = 0 \\
\dfrac{R_{W,i}}{MW_W}x_{N,W}\Psi_{NO,i} - W^{out}_{NO,G,i} = 0 \\
\begin{cases}
\dfrac{R_{W,1}}{MW_W}\dfrac{x_{H,W} - x_{Cl,W}}{2} - W^{out}_{,H_2OG,1} + \dfrac{F_W\omega_{H_2O,0}}{MW_{H_2O}} = 0 \\
\dfrac{R_{W,i}}{MW_W}\dfrac{x_{H,W} - x_{Cl,W}}{2} - W^{out}_{H_2O,G,i} = 0 \quad i = 2,\ldots,NG
\end{cases}
\end{cases} \qquad (17)$$

Model equations

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$$M_{W,F,i} = \rho_W \Delta x_G w_G s_{B,i} \qquad (18)$$

$$F_{W,i}^{out} = M_{W,F,i} f_{W,i} N_{GS,i} \qquad (19)$$

$$F_{I,i}^{out} = M_{W,F,i}(1 - f_{W,i}) N_{GS,i} \qquad (20)$$

$$f(z) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(z - z_0)^2}{2\sigma^2}\right) \qquad (21)$$

$$F_{W,i}^{out} = M_{W,F,i} f_{W,i} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{[t - (t_{0,i} + \theta_i)]^2}{2\sigma^2}\right) \qquad (22)$$

$$F_{I,i}^{out} = M_{W,F,i}(1 - f_{W,i}) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{[t - (t_{0,i} + \theta_i)]^2}{2\sigma^2}\right) \qquad (23)$$

$$h_i \cong \frac{0.5}{N_{GS,i}} \qquad (24)$$

$$\frac{\int_0^{3600} M_{W,F,0} \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{[t - (t_{0,k} + \theta)]^2}{2\sigma^2}\right) dt}{3600} \cong \bar{F}_W \quad t_{0,k} < t < t_{0,k} + \frac{1}{N_{GS,0}} \qquad (25)$$

$$\theta = k\sqrt{\sigma^2} \qquad (26)$$

$$Z = -\frac{t_0 - (t_0 + \theta)}{\sqrt{\sigma^2}}$$

$$\theta = \bar{Z}\sqrt{\sigma^2} \Rightarrow k = \bar{Z} = -\frac{t_0 - (t_0 + \theta)}{\sqrt{\sigma^2}} \qquad (27)$$

Model equations

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$$W_{Hom,j}^{in} = \sum_{i=1}^{NG} W_{G,j,i}^{out} \quad j = \mathrm{CO,SO_2,HCl,NO,H_2O}$$

$$W_{Hom,O_2}^{in} = \sum_{i=1}^{NG} W_{G,O_2,i}^{out} + (W_{A,S} + W_{A,LS})x_{O_2,A} \qquad (28)$$

$$W_{Hom,N_2} = \sum_{i=1}^{NG} W_{G,N_2,i}^{out} + (W_{A,S} + W_{A,LS})x_{N_2,A}$$

$$x_{Hom,j}^{in} = \frac{W_{Hom,j}^{in}}{\sum_{j=1}^{NC} W_{Hom,j}^{in}} \quad j = \mathrm{O_2,N_2,CO,SO_2,HCl,NO,H_2O}$$

$$\mathrm{CO} + \frac{1}{2}\mathrm{O_2} \rightarrow \mathrm{CO_2} \qquad (29)$$

$$\mathrm{N_2} + \mathrm{O_2} \rightarrow 2\mathrm{NO} \qquad (30)$$

$$\frac{dc_{CO}}{dt} = 1.8 \times 10^{10} c_{CO}\sqrt{c_{O_2}}\exp\left(-\frac{25000}{RT}\right) \qquad (31)$$

$$\begin{cases} \mathrm{O} + \mathrm{N_2} \rightarrow \mathrm{N} + \mathrm{NO} \\ \mathrm{N} + \mathrm{O_2} \rightarrow \mathrm{O} + \mathrm{NO} \\ \mathrm{N} + \mathrm{OH} \rightarrow \mathrm{H} + \mathrm{NO} \end{cases} \qquad (32)$$

$$\frac{dn_{Hom,CO_2}}{dt} = -(W_{Hom}^{out} + W_L^{out})x_{Hom,CO_2}^{out} + R_{Hom,CO_2}V_{Hom} + W_{Hom,CO_2}^{in}$$

$$\frac{dn_{Hom,j}}{dt} = -(W_{Hom}^{out} + W_L^{out})x_{Hom,j}^{out} + W_{Hom,j}^{in} \quad j = \mathrm{SO_2,H_2O,HCl}$$

$$\frac{dn_{Hom,N_2}}{dt} = -(W_{Hom}^{out} + W_L^{out})x_{Hom,N_2}^{out} - \frac{1}{2}R_{Hom,NO}V_{Hom} + W_{Hom,N_2}^{in} \qquad (33)$$

$$\frac{dn_{Hom,O_2}}{dt} = -(W_{Hom}^{out} + W_L^{out})x_{Hom,O_2}^{out} - \frac{1}{2}R_{Hom,NO}V_{Hom} - \frac{1}{2}R_{Hom,CO_2}V_{Hom} + W_{Hom,O_2}^{in}$$

$$\frac{dn_{Hom,CO^*}}{dt} = -(W_{Hom}^{out} + W_L^{out})x_{Hom,CO^*}^{out} - R_{Hom,CO_2}V_{Hom} + W_{Hom,CO}^{in}(1 - f_{bypass})$$

$$\frac{dn_{Hom,CO^{**}}}{dt} = -(W_{Hom}^{out} + W_L^{out})x_{Hom,CO^{**}}^{out} + W_{Hom,CO}^{in}f_{bypass}$$

$$x_{Hom,CO}^{out} = \frac{n_{Hom,CO^*} + n_{Hom,CO^{**}}}{\sum_{j=1}^{NC+1} n_{Hom,j}}$$

Model equations

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$$P_1 - P_2 = \gamma \rho_1 \frac{{v_1}^2}{2} \tag{34}$$

$$W_L^{in} = \bar{k}\sqrt{\Delta P} \tag{35}$$

$$W_L^{out} = \bar{k}\frac{c_{tot,g}}{c_{tot,amb}}\sqrt{\frac{\rho_{amb}}{\rho_g}}\sqrt{\Delta P} \tag{36}$$

$$W_{Hom}^{out} = \sqrt{\frac{2P_{PK}}{\gamma}\cdot\frac{|P_{PK} - P_{PC}|}{MW_g RT}}A^{out} \tag{37}$$

$$\frac{dU}{dt} = \dot{H}_{in} - \dot{H}_{out} - \dot{Q}_{disp} - \dot{Q}_{V,PK} \tag{38}$$

$$U = \sum_{i=1}^{NC} n_i U_{g,i} + M_S U_S$$
$$\dot{H} = \sum_{i=1}^{NC} W_i H_{g,i} + \dot{M}_S H_S \tag{39}$$

$$\tilde{c}_p = \tilde{c}_v + R$$
$$H_{g,i} = U_{g,i} + P\tilde{V} = U_{g,i} + RT \tag{40}$$

$$\hat{c}_{v,S} \approx \hat{c}_{p,S} \Rightarrow U_S \approx H_S \tag{41}$$

$$\left(M_S\hat{c}_{p,S} + \sum_{i=1}^{NC} n_i \tilde{c}_{v,i}\right)\frac{dT}{dt} = -\sum_{i=1}^{NC}\left(W_{Hom,i}^{in}\int_{T^{in}}^{T}\tilde{c}_{p,i}\,dT\right) - \dot{M}_S^{in}\int_{T^{in}}^{T}\hat{c}_{p,s}\,dT - \sum_{j=1}^{NR}\Delta H_j^R(T)R_j(T) + R_W WLHC + RT\frac{d}{dt}\left(\sum_{i=1}^{NC} n_i\right) - \dot{Q}_{disp,PK} - \dot{Q}_{V,PK} \tag{42}$$

Model equations

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$$\sum_{i=1}^{NC}(W_{Hom,i}^{in}\int_{T^{in}}^{T}\tilde{c}_{p,i}\,dT) = (W_{A,PK} + W_{A,L})\int_{T_{amb}}^{T}\tilde{c}_{p,A}\,dT \quad (43)$$

$$\dot{M}_S^{in}\int_{T^{in}}^{T}\hat{c}_{p,S}\,dT = F_W^{in}(1-\omega_{H_2O})\hat{c}_{p,S}(T-T^{in}) + \frac{F_W^{in}\omega_{H_2O}}{MW_{H_2O}}\int_{T_{eb}}^{T}\tilde{c}_{p,H_2O}^{v}\,dT \quad (44)$$

$$\sum_{j=1}^{NR}\Delta H_j^R(T)R_j(T) = \Delta H_{CO\rightarrow CO_2}^R R_{CO_2} + \Delta H_{N_2\rightarrow NO}^R R_{NO} \quad (45)$$

$$R_W WLHC^* = \sum_{i=1}^{NG}\frac{R_{W,i}}{1-\omega_I-\omega_{H_2O}}\left(WLHC + \Delta H_{CO\rightarrow CO_2}^R\frac{\omega_C}{MW_C} + \Delta H_{N_2\rightarrow NO}^R\frac{\omega_N}{MW_N}(1-\varphi_{NO,i})\right) \quad (46)$$

$$RT\frac{d}{dt}\sum_{i=1}^{NC}n_i \quad (47)$$

$$\dot{Q}_{rad} = \sigma(A_{G\rightarrow W}{T_g}^4 - A_{W\rightarrow G}{T_W}^4) \quad (48)$$

$$A_{G\rightarrow W}T_g \simeq A_{W\rightarrow G}T_W \quad (49)$$

$$\dot{Q}_{rad} = \sigma A_{G\rightarrow W}({T_g}^4 - {T_W}^4)\frac{1-k^3}{1-k^4} \qquad k = \frac{T_W}{T_g} \quad (50)$$

$$A_{G\rightarrow W} = \frac{A_{GW}}{\frac{1}{\epsilon_g}+\frac{1}{\epsilon_R}-1} \quad (51)$$

$$\dot{Q}_{conv} = \sigma A_{G\rightarrow W}({T_g}^4 - {T_W}^4)A_{GW}\frac{h_{int}}{4\sigma {T_{G\rightarrow W}}^3}$$

$$T_{G\rightarrow W} = \frac{T_g + T_W}{2} \quad (52)$$

Model equations

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$$\dot{Q}_{V,PK} = \beta_{PK}\dot{Q}_{disp,PK} \qquad (53)$$

$$\frac{\partial T}{\partial t} = k_W \frac{\partial^2 T}{\partial z^2} \qquad (54)$$

$$\rho_i \hat{c}^W_{p,i} V_i \frac{dT_i}{dt} = \dot{q}_i^{in} A_i^{int} - \dot{q}_i^{out} A_i^{out} \quad i = 1,\ldots,NL \qquad (55)$$

$$T(z_i) = T_i \qquad T(z_{i+1}) = T_{i+1} \qquad (56)$$

$$\dot{q}_i^{out} = -k_W \left.\frac{dT}{dz}\right|_{i+1/2} = k_W \frac{T_i - T_{i+1}}{z_{i+1} - z_i} \qquad (57)$$

$$\dot{q}_{RI} = \frac{T^R_{NRL} - T^I_1}{\dfrac{s_R}{2k_R} + \dfrac{s_I}{2k_I}} \qquad (58)$$

$$h_{int}(T_{g,PK} - T_W^{int}) = \frac{2k_R}{s_R}(T_W^{int} - T_1^R) \qquad (59)$$

$$h_{ext}(T_S^{ext} - T_{amb}) = \frac{2k_I}{s_I}(T^I_{NIL} - T_S^{ext}) \qquad (60)$$

$$h_{ext} = \frac{Nu k_A}{L} + 4\sigma\epsilon_{met}{T_{met}}^3 \qquad T_{met} = \frac{T_{amb} + T_S^{ext}}{2} \qquad (61)$$

$$\frac{dn_{CO_2}}{dt} = -W^{out}_{PC} x^{out}_{CO_2,PC} + R_{CO_2} V_{PC} + W^{in}_{PC} x^{in}_{CO_2,PC} \qquad (62)$$

$$\frac{dn_{SO_2}}{dt} = -W^{out}_{PC} x^{out}_{SO_2,PC} + W^{in}_{PC} x^{in}_{SO_2,PC} \qquad (63)$$

Model equations

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$$\frac{dn_{H_2O}}{dt} = -W_{PC}^{out} x_{H_2O,PC}^{out} + W_{PC}^{in} x_{H_2O,PC}^{in} \qquad (64)$$

$$\frac{dn_{HCl}}{dt} = -W_{PC}^{out} x_{HCl,PC}^{out} + W_{PC}^{in} x_{HCl,PC}^{in} \qquad (65)$$

$$\frac{dn_{N_2}}{dt} = -W_{PC}^{out} x_{N_2,PC}^{out} - \frac{1}{2} R_{NO} V_{PC} + W_{PC}^{in} x_{N_2,PC}^{in} \qquad (66)$$

$$\frac{dn_{O_2}}{dt} = -W_{PC}^{out} x_{O_2,PC}^{out} - \frac{1}{2} R_{NO} V_{PC} - \frac{1}{2} R_{CO_2} V_{PC} + W_{PC}^{in} x_{O_2,PC}^{in} \qquad (67)$$

$$\frac{dn_{CO}}{dt} = -W_{PC}^{out} x_{CO,PC}^{out} - R_{CO_2} V_{PC} + W_{PC}^{in} x_{CO,PC}^{in} \qquad (68)$$

$$\frac{dn_{NO}}{dt} = -W_{PC}^{out} x_{NO,PC}^{out} + R_{NO} V_{PC} + W_{PC}^{in} x_{NO,PC}^{in} \qquad (69)$$

$$P_{PC} - P_{PK} = \Delta P_{dis} + \Delta P_{loc} \qquad (70)$$

$$\Delta P_{dis} = \frac{2 f_D \rho_g {v_g}^2 L_{PC}}{D_{h,PC}} \qquad (71)$$

$$f_D = \frac{1}{\left[4 \log\left(0.27 \frac{\epsilon}{D_h} + \left(\frac{7}{Re}\right)^{0.9}\right)\right]^2} \qquad (72)$$

$$\Delta P_{loc} = \gamma \frac{\rho_g {v_g}^2}{2} \qquad (73)$$

$$\frac{dU}{dt} = \dot{H}_{in} - \dot{H}_{out} - \dot{Q}_{disp,PC} - \dot{Q}_{V,PC} \qquad (74)$$

Model equations

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$$\left(\sum_{i=1}^{NC} n_i \tilde{c}_{v,i}\right)\frac{dT}{dt} = -\sum_{i=1}^{NC}\left(W_{i,PC}^{in}\int_{T^{in}}^{T} \tilde{c}_{p,i}\, dT\right) - \sum_{j=1}^{NR}\Delta H_j^R(T)R_j(T) + RT\frac{d}{dt}\left(\sum_{i=1}^{NC} n_i\right) - \dot{Q}_{disp,PC} - \dot{Q}_{V,PC} \quad (75)$$

$$\frac{dM_L}{dt} = \dot{G}_{H_2O}^{in} - \dot{G}_{L\to V} \quad (76)$$

$$\frac{dM_V}{dt} = \dot{G}_{L\to V} - \dot{G}_V \quad (77)$$

$$\Delta P_{SH} = 2 f_D \rho_V v_V^2 \frac{L_{SH}}{D_{h,SH}} \quad (78)$$

$$\dot{G}_V = \rho_V A_V v_V \quad (79)$$

$$\dot{G}_V = A_V\sqrt{\frac{\rho_V D_{h,SH}\Delta P_{SH}}{2 f_D L_{SH}}} \quad (80)$$

$$P = \exp\left(73.649 - \frac{7258.2}{T} - 7.3037\log(T) + 4.1653\times 10^{-6}\cdot T^2\right) \quad (81)$$

$$\dot{G}_{L\to V} = \frac{\dot{G}_{abs}^{B} - \dot{G}_{H_2O}^{in}\hat{c}_{p,L}(T_{eq} - T_{H_2O}^{in})}{\Delta H_{ev}(T_{eq})} \quad (82)$$

$$\Delta H_{ev}(T) = \Delta H_{ev}(T_{ref})\left(\frac{1 - T_R}{1 - T_{R,ref}}\right) \quad T_R = \frac{T}{T_C} \quad (83)$$

$$\dot{G}_{H_2O}^{in} = \dot{G}_{H_2O}^{in,0} + k_p\left(\epsilon_c + \frac{1}{\tau_I}\int_0^t \epsilon_c\, dt\right) \quad (84)$$

Model equations

Manca D., M. Rovaglio, Ind. Eng. Chem. Res. 2005, *44*, 3159-3177

$$\frac{dn_{g,B}}{dt} = W_{g,B}^{in} - W_{g,B}^{out} \quad (85)$$

$$W_{g,B}^{out} = \frac{\rho_g A_B v_g}{MW_{g,B}^{mix}} \quad (86)$$

$$\dot{H}_B^{in} - \dot{H}_B^{out} - \dot{Q}_{V,B} = 0 \quad (87)$$

$$m = \frac{1}{1 + \frac{155.3\sqrt{q}}{h_g}}$$

$$q = \frac{\dot{H}_B^{in}}{A_B^{exc}} \quad h_g = \int_{T_{ref}}^{T_g^{in}} \hat{c}_{p,g}^{mix}\, dT \quad (88)$$

$$\dot{Q}_{V,B} = m\dot{H}_{in} = m\dot{W}_{g,B}\int_{T_{ref}}^{T^{in}{g,B}} \tilde{c}_{p,g}^{mix}\, dT \quad (89)$$

$$\left(\frac{T_{g,B}^{rad,out}}{100}\right)^4 + 239 \cdot q \cdot \left(\frac{T_{g,B}^{rad,out}}{T_{ref} + \frac{h_g}{\hat{c}_{p,g}^{mix}}} - 1\right) = 0 \quad (90)$$

$$\dot{Q}_{V,B} = m\dot{H}_{in} = \frac{1}{1 + \frac{\frac{461.9}{h_g^{0.15}}\sqrt{q}}{h_g}} W_{g,B}^{in} \int_{T_{ref}}^{T_{g,B}^{in}} \tilde{c}_{p,g}^{mix}\, dT \quad (91)$$

$$T_{g,B}^{rad,out} = T_{ref} + \frac{1000}{\frac{2.52}{\sqrt{q}} h_g^{0.15} + \frac{\hat{c}_{p,g}^{mix}}{h_g}} \quad (92)$$

$$W_{g,SH}^{in} = W_{g,B}^{out} + W_{L,SH} \quad (93)$$

$$W_{g,SH}^{in} \int_{T_{ref}}^{T_{g,SH}^{in}} \tilde{c}_{p,g}^{mix}\, dT = W_{g,B}^{out} \int_{T_{ref}}^{T_{g,B}^{out}} \tilde{c}_{p,g}^{mix}\, dT + W_{L,SH} \int_{T_{ref}}^{T_{L,SH}} \tilde{c}_{p,A}\, dT \quad (94)$$

Model equations

Manca D., M. Rovaglio, Ind. Eng. Chem. Res. 2005, *44*, 3159-3177

$$u_m = \frac{u_{in} s_t}{s_t - d_t} \text{ if } 2(s_d - d_t) \geq (s_t - d_t)$$
$$u_m = \frac{u_{in} s_t}{2(s_d - d_t)} \text{ if } 2(s_d - d_t) < (s_t - d_t) \qquad (95)$$

$$h_{rad} = \sigma A_{G \to T} \left(\frac{\bar{T}_H + \bar{T}_C}{2} \right)^3 \qquad (96)$$

$$U = \frac{1}{\frac{1}{h_{int}} + \frac{1}{h_{conv} + h_{rad}} + f_{fou}} \qquad (97)$$

$$\begin{cases} \eta = \dfrac{1 - \exp(-NTU(1-r))}{1 - r \cdot \exp(-NTU(1-r))} & \text{if } r \leq 0.98 \\ \eta = \dfrac{NTU}{1 + NTU} & \text{if } 0.98 < r \leq 1 \end{cases}$$

$$r = \frac{c_{MIN}}{c_{MAX}}$$
$$\begin{cases} c_{MIN} = \text{Min}\{F_H \hat{c}_{p,H}, F_C \hat{c}_{p,C}\} \\ c_{MAX} = \text{Max}\{F_H \hat{c}_{p,H}, F_C \hat{c}_{p,C}\} \end{cases} \qquad (98)$$
$$NTU = \frac{UA_{GT}}{c_{MIN}}$$

$$T_C^{out} = T_C^{in} + \eta (T_{H,eff}^{in} - T_C^{in})$$
$$T_H^{out} = T_{H,eff}^{in} - \eta r (T_{H,eff}^{in} - T_C^{in}) \qquad (99)$$

$$T_C^{out} = T_C^{in} + \eta r (T_{H,eff}^{in} - T_C^{in})$$
$$T_H^{out} = T_{H,eff}^{in} - \eta (T_{H,eff}^{in} - T_C^{in}) \qquad (100)$$

$$\Delta P_{exc} = f_D \rho_g {v_{g,MAX}}^2 n_T \qquad (101)$$

Model dimensions in terms of DAEs

	Material balances	*Energy balances*	*Momentum balances*	***Total***
Primary kinl	**55**	**29**	**2**	**86**
Postcombustion chamber	**10**	**15**	**1**	**26**
Heat recovery section	**6**	**9**	**5**	**20**

Grand total = 132 DAE system

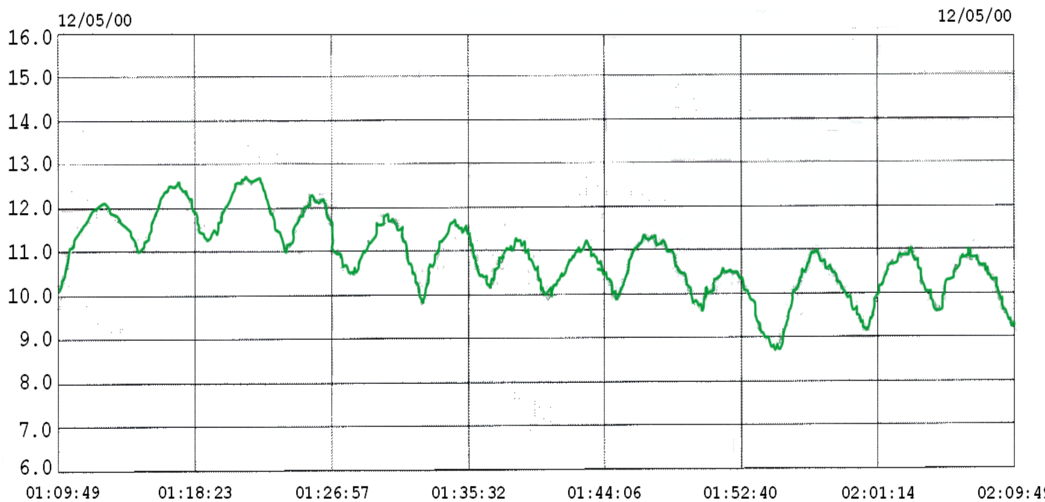
Some significant input process variables

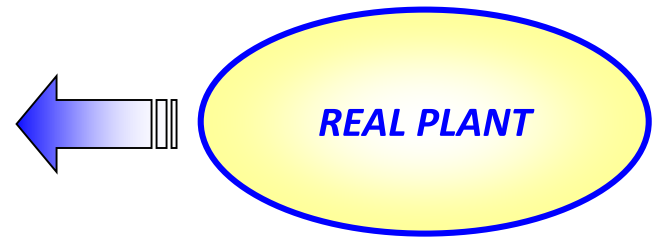
Number of feed-strokes [1/h]	40
Inlet waste flowrate [kg/h]	4,000
Number of strokes to the first grate [1/h]	23
Number of strokes to the second grate [1/h]	20
Number of strokes to the third grate [1/h]	13
Number of strokes to the fourth grate [1/h]	23
Primary air flowrate [Nm3/h]	12,500
Secondary air flowrate [Nm3/h]	6,500

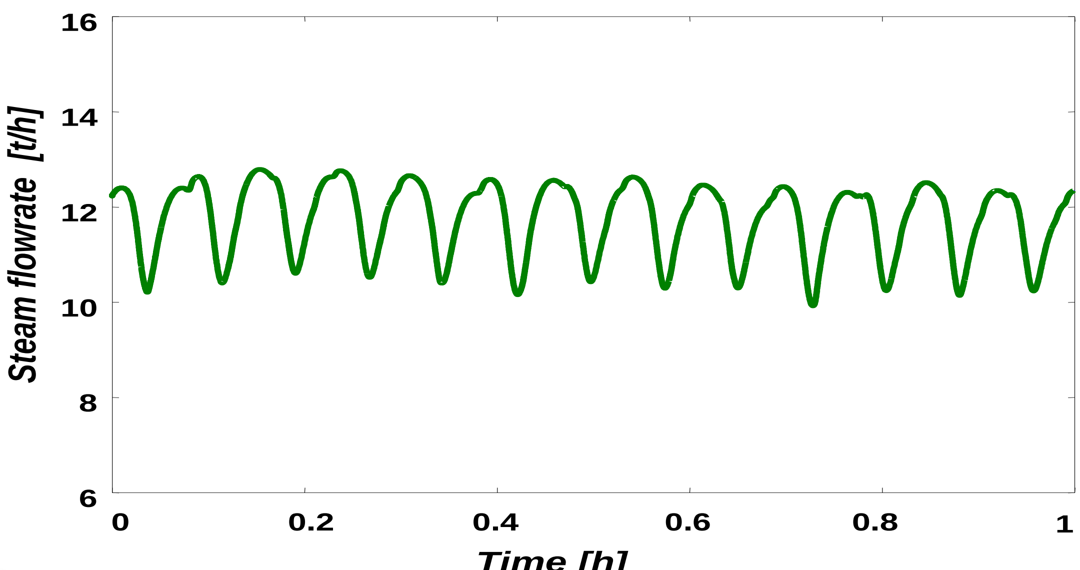
Some significant output process variables

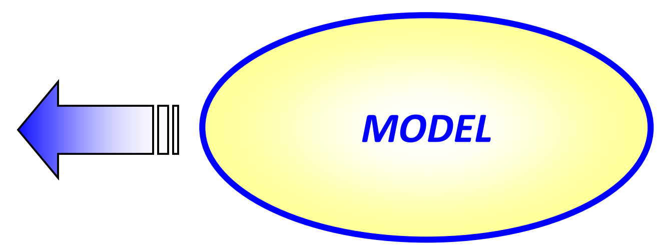
Outlet smokes temperature from the primary kiln [°C]	1,035
Outlet smokes temperature from the postcombustion chamber [°C]	1,115
Outlet oxygen molar fraction from the postcombustion chamber [%]	8.5
Outlet CO content from the postcombustion chamber [mg/Nm3]	11
Steam flowrate [t/h]	12

Model validation

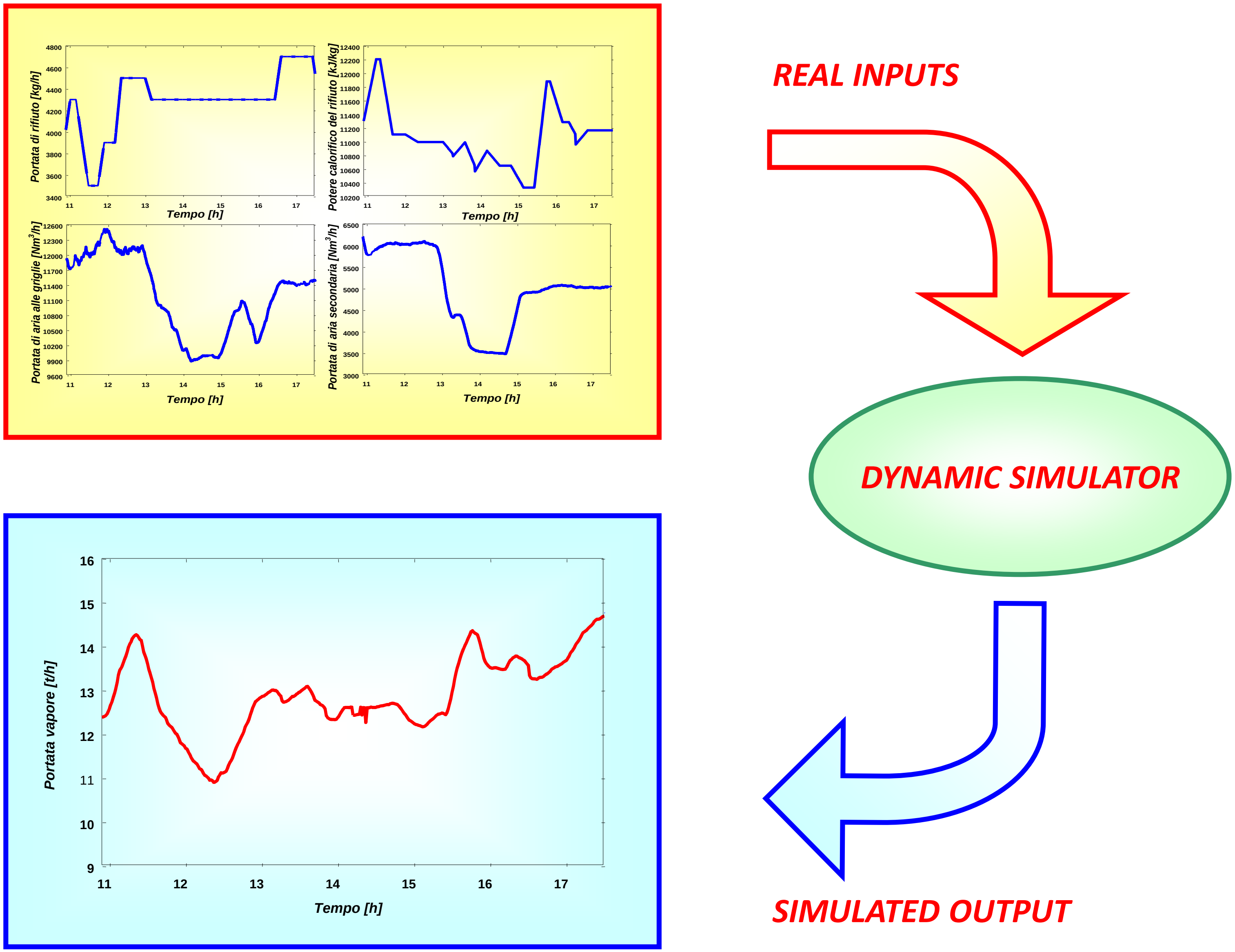






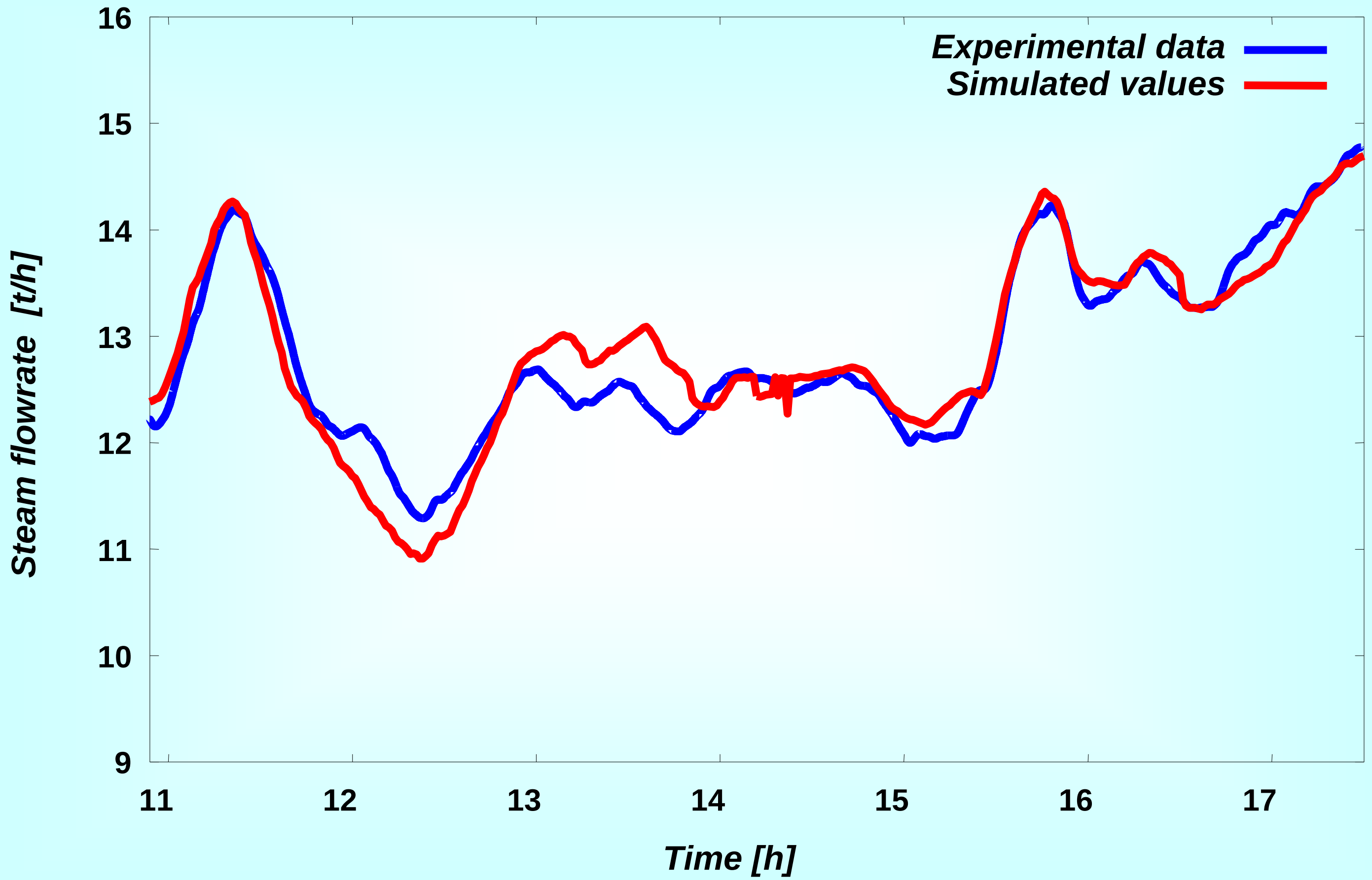


Model validation

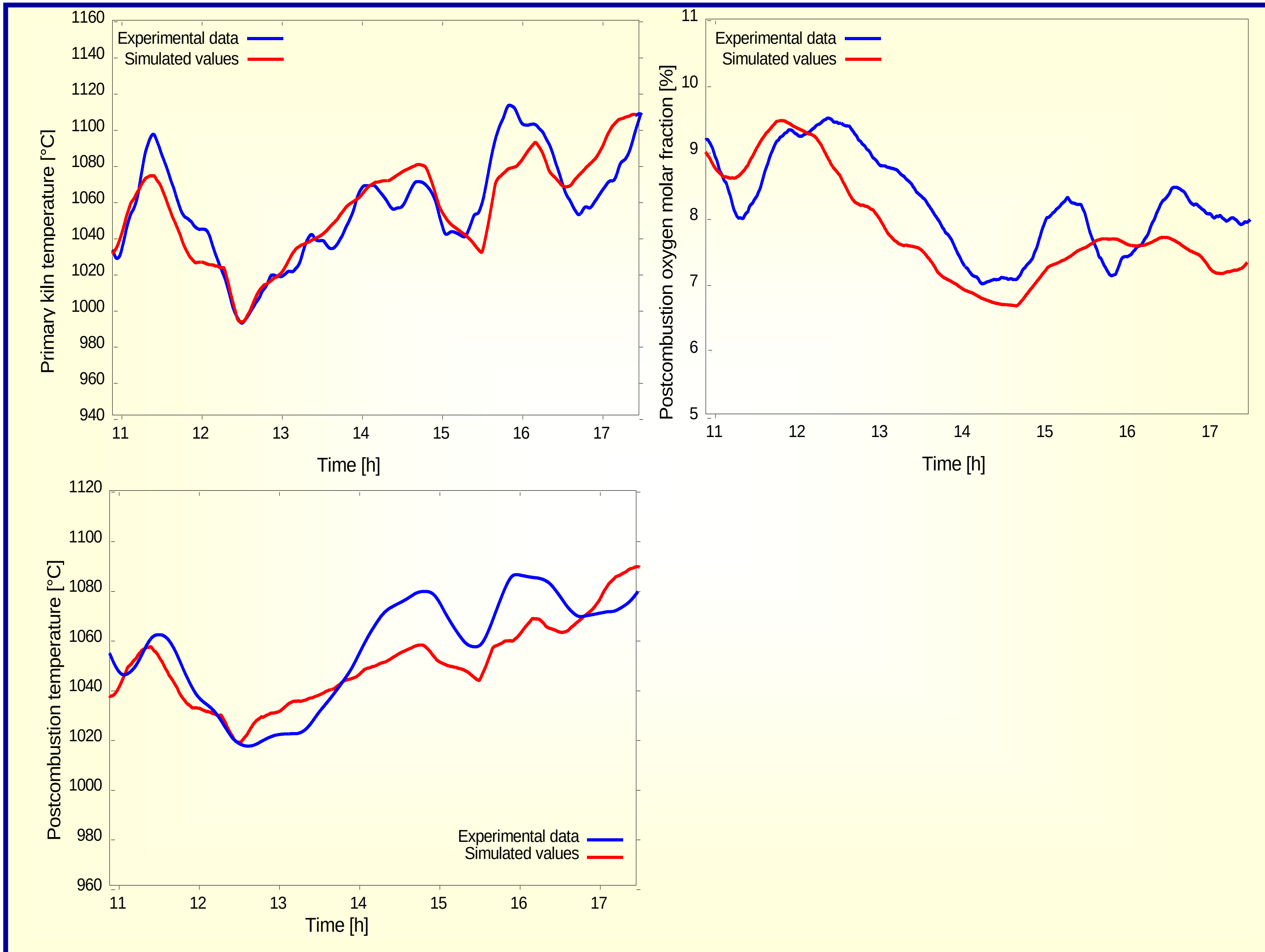


Model validation

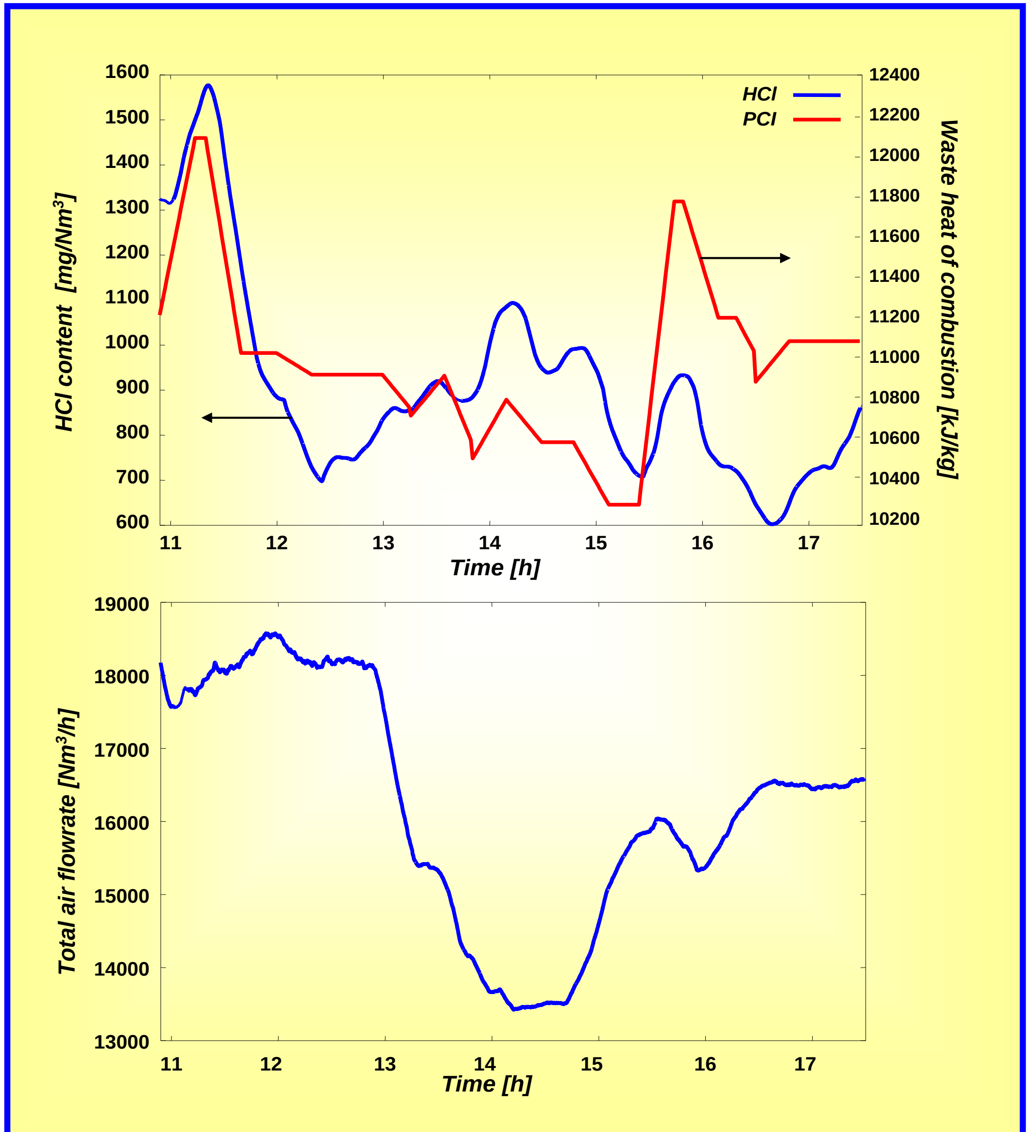
Real plant ⟷ Model



On line model validation



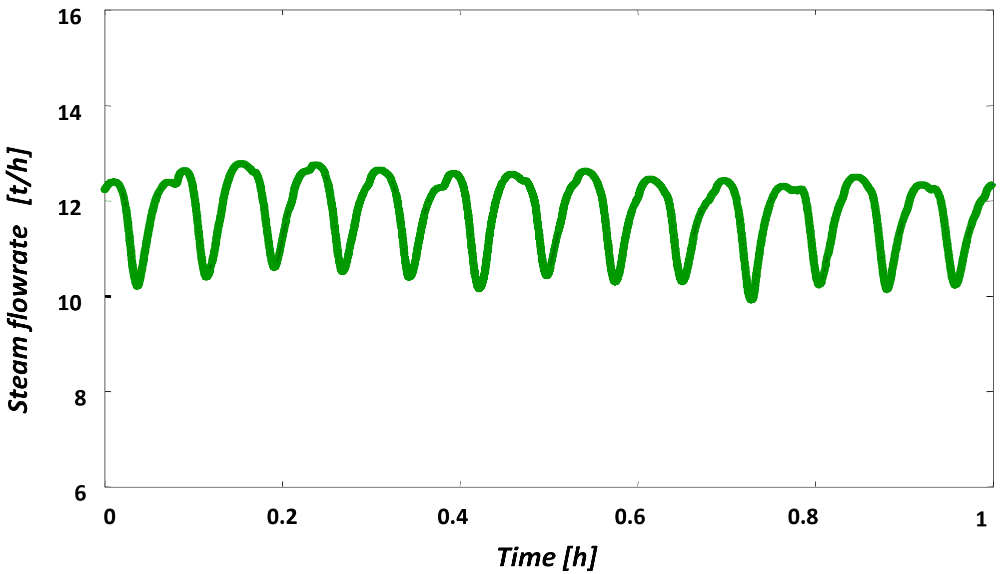
Inference of heat of combustion and HCl



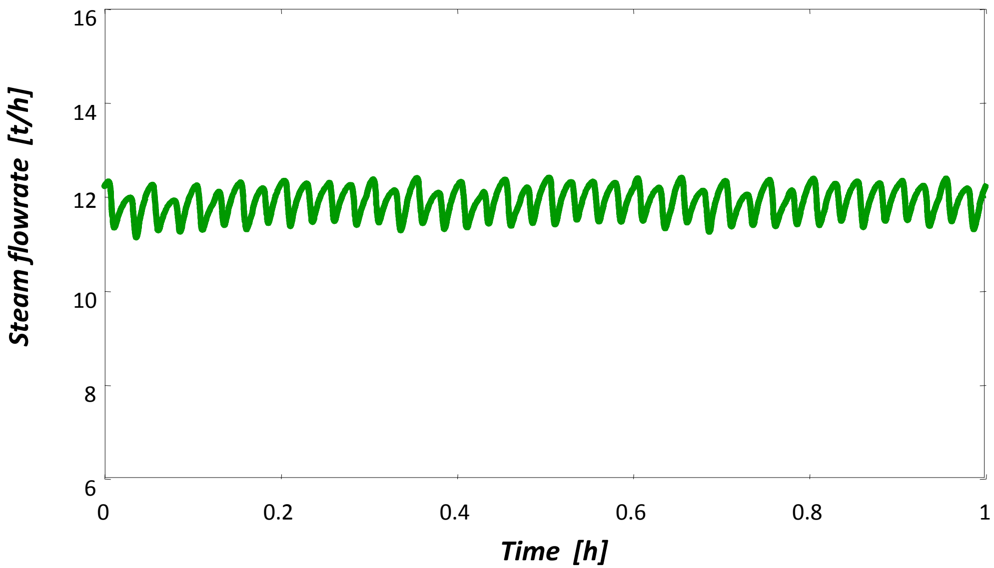
Inferred waste heat of combustion

Experimental trend of the total air flowrate

Dynamic response to different forcing actions



3 x N



1 x 3N

Multivariable model based control

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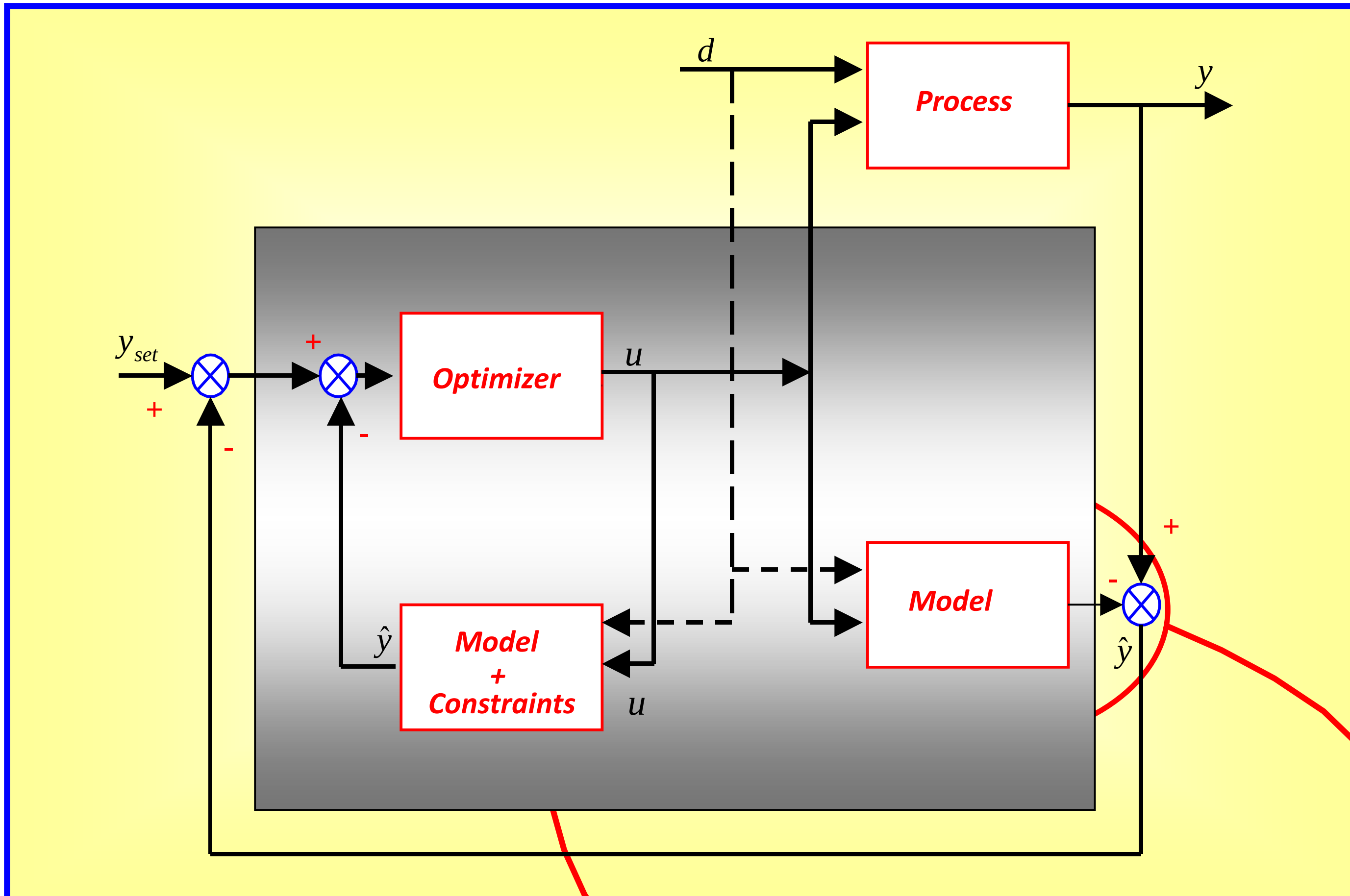
MPC – Implementation scheme



Use of a model for predictive pursposes

MPC – Objective function

Optimization problem to be solved:

$$\min_{\hat{u}(k)....\hat{u}(k+h_c-1)} f_{obj}$$

$$f_{obj} = \left\{ \sum_{j=k+1}^{k+h_p} \left[\omega_y \cdot \left(\hat{e}_y(j) \right)^2 + PF_y(j) \right] + \sum_{i=k}^{k+h_p-1} \left[\omega_u \cdot \left(\Delta\hat{u}(i) \right)^2 + PF_u(i) \right] \right\}$$

$$\hat{e}_y(j) = \frac{\left[\hat{y}(j) + \delta(k) \right] - \hat{y}_{SET}(j)}{\hat{y}_{SET}(j)}; \qquad \delta(k) = y(k) - \hat{y}(k)$$

$$PF_y(j) = \left\{ \max\left[0, \left(\frac{\hat{y}(j) - \hat{y}_{MAX}}{\hat{y}_{MAX}} \right) \right] \right\}^2 + \left\{ \min\left[0, \left(\frac{\hat{y}(j) - \hat{y}_{MIN}}{\hat{y}_{MIN}} \right) \right] \right\}^2$$

$$\Delta\hat{u}(i) = \frac{\hat{u}(i) - \hat{u}(i-1)}{\hat{u}(i-1)}$$

$$PF_u(i) = \left\{ \max\left[0, \left(\frac{\hat{u}(i) - \hat{u}_{MAX}}{\hat{u}_{MAX}} \right) \right] \right\}^2 + \left\{ \min\left[0, \left(\frac{\hat{u}(i) - \hat{u}_{MIN}}{\hat{u}_{MIN}} \right) \right] \right\}^2$$

MPC – Controlled variables (CV)

CONTROLLED VARIABLES (CV)

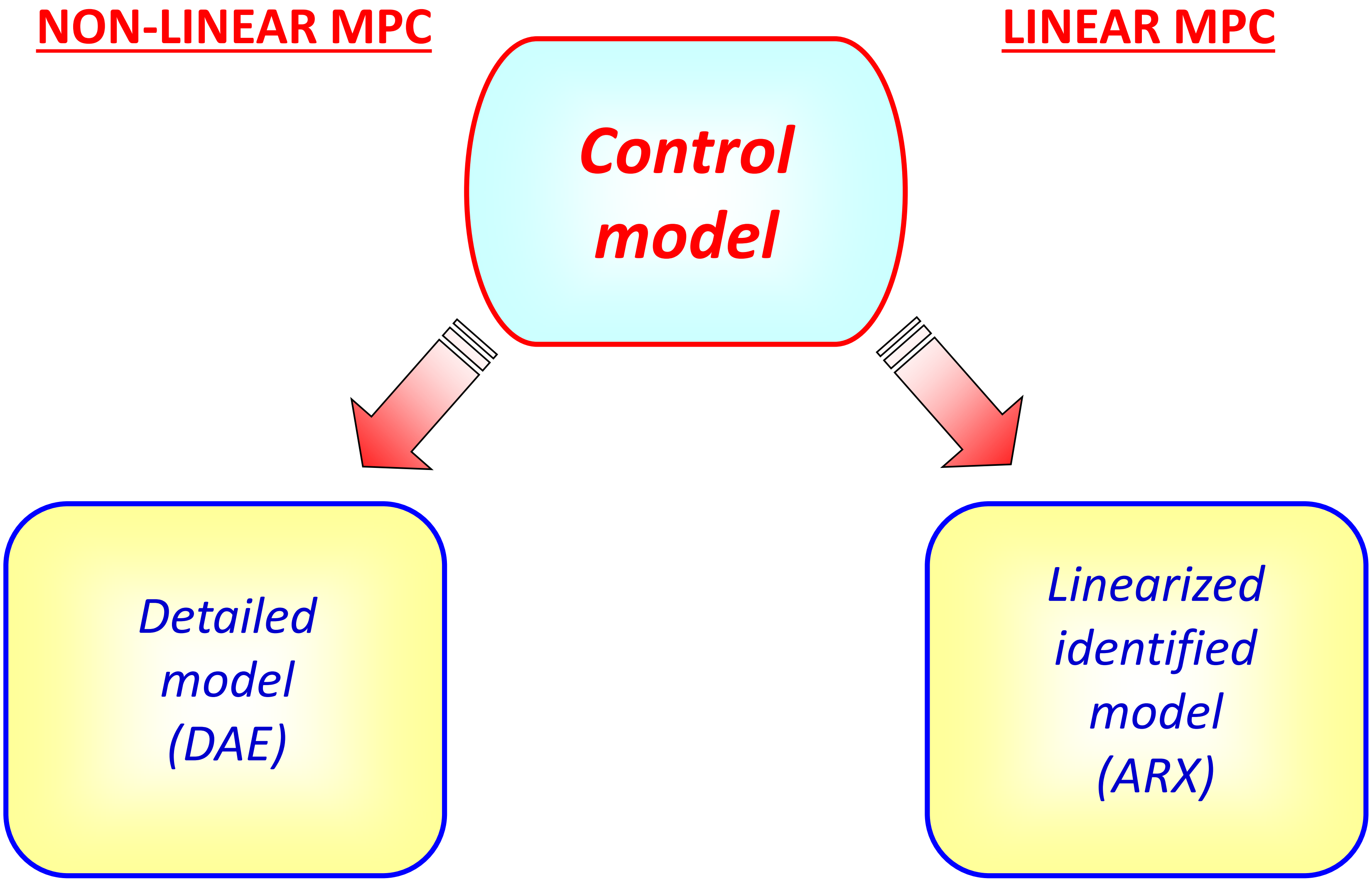
- ***Steam flowrate (set point)***
- ***Primary kiln temperature (upper bound)***
- ***Postcombustion chamber temperature (upper bound)***
- ***Postcombustion chamber temperature(lower bound)***
- ***Volumetric O_2 fraction in the postcombustion chamber (lower bound)***
- ***CO content in the postcombustion chamber (upper bound)***

MPC – Manipulated variables (MV)

MANIPULATED VARIABLES (MV)

- ***Strokes number to the feed grate***
- ***Primary air flowrate (to the grates)***
- ***Secondary air flowrate to the kiln***
- ***Strokes number to the first moving grate***
- ***Strokes number to the second moving grate***

MPC – Control model

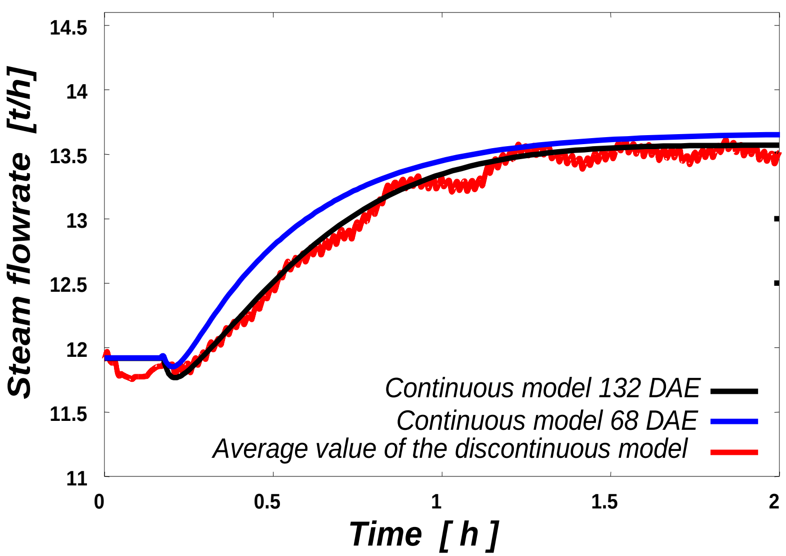


NL MPC – Continuous simplified model

Averaged flowrate ⇒

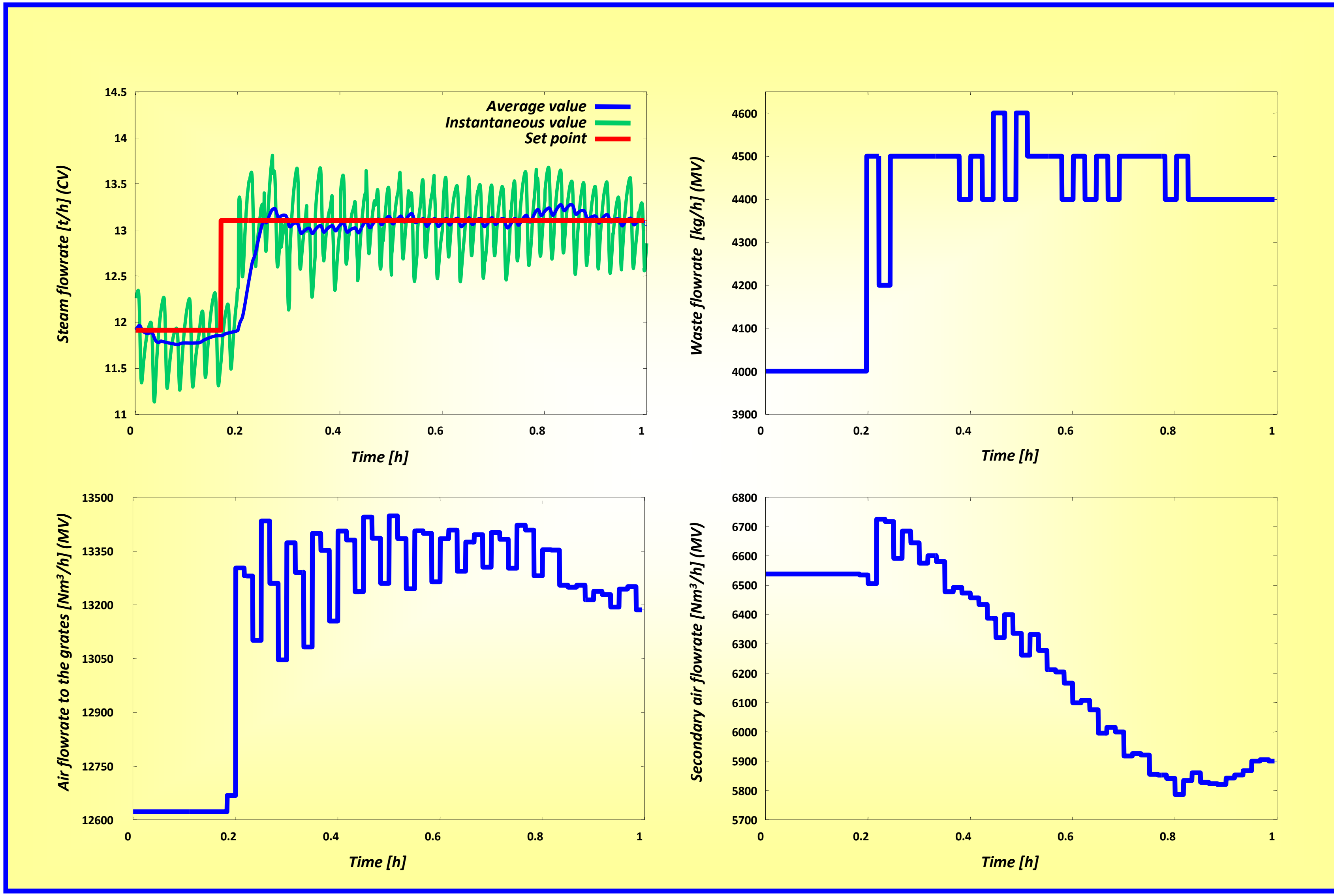
$$\bar{F}_{waste} = \Delta m \cdot \frac{\displaystyle\int_{t_o}^{t_o+\frac{3600}{N_{CG}}} \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma^2}} \cdot \exp\left(-\frac{(t-(t_0+\theta))^2}{2 \cdot \sigma^2}\right)}{\dfrac{3600}{N_{CG}}}$$

CPU time for a 1 min simulation interval	**10% disturbance on the waste flowrate**
Discontinuous model (132 DAE)	6.02 s
Continuous model (132 DAE)	2.26 s
Simplified continuous model (68 DAE)	0.54 s

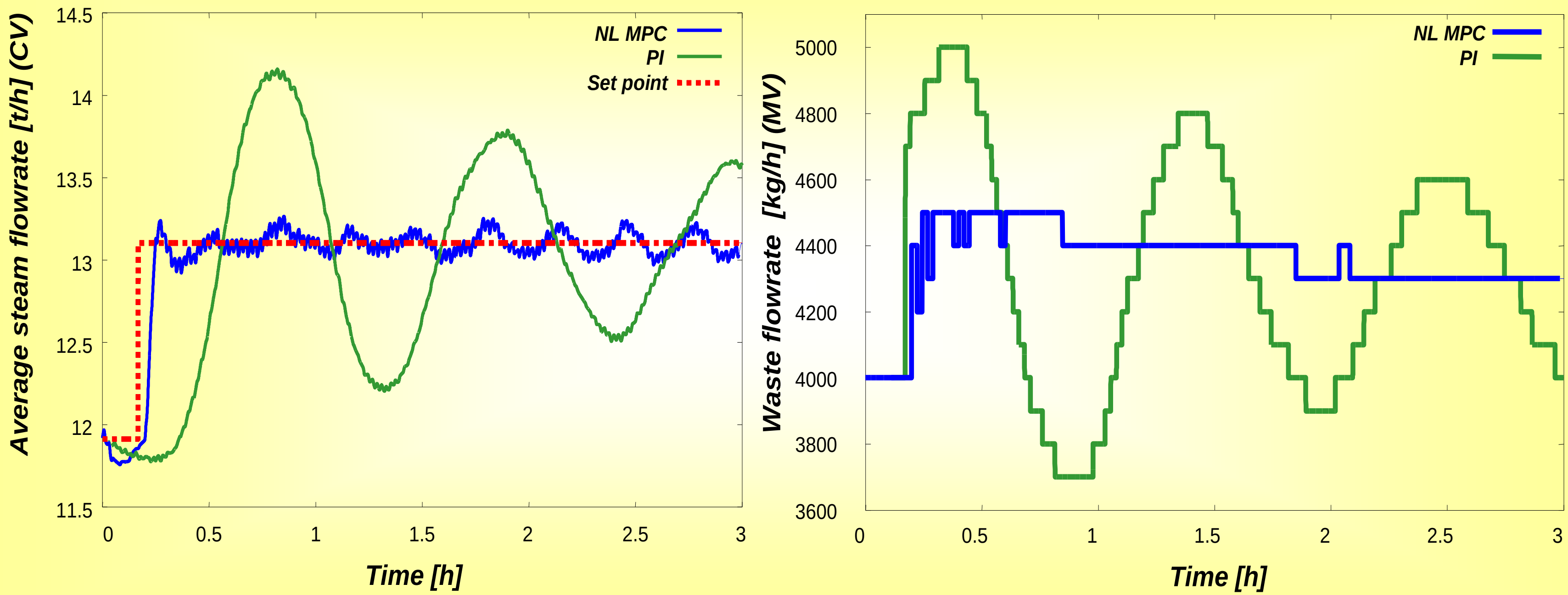


Same simulated average values but significantly reduced CPU times

NL MPC – Servo problem



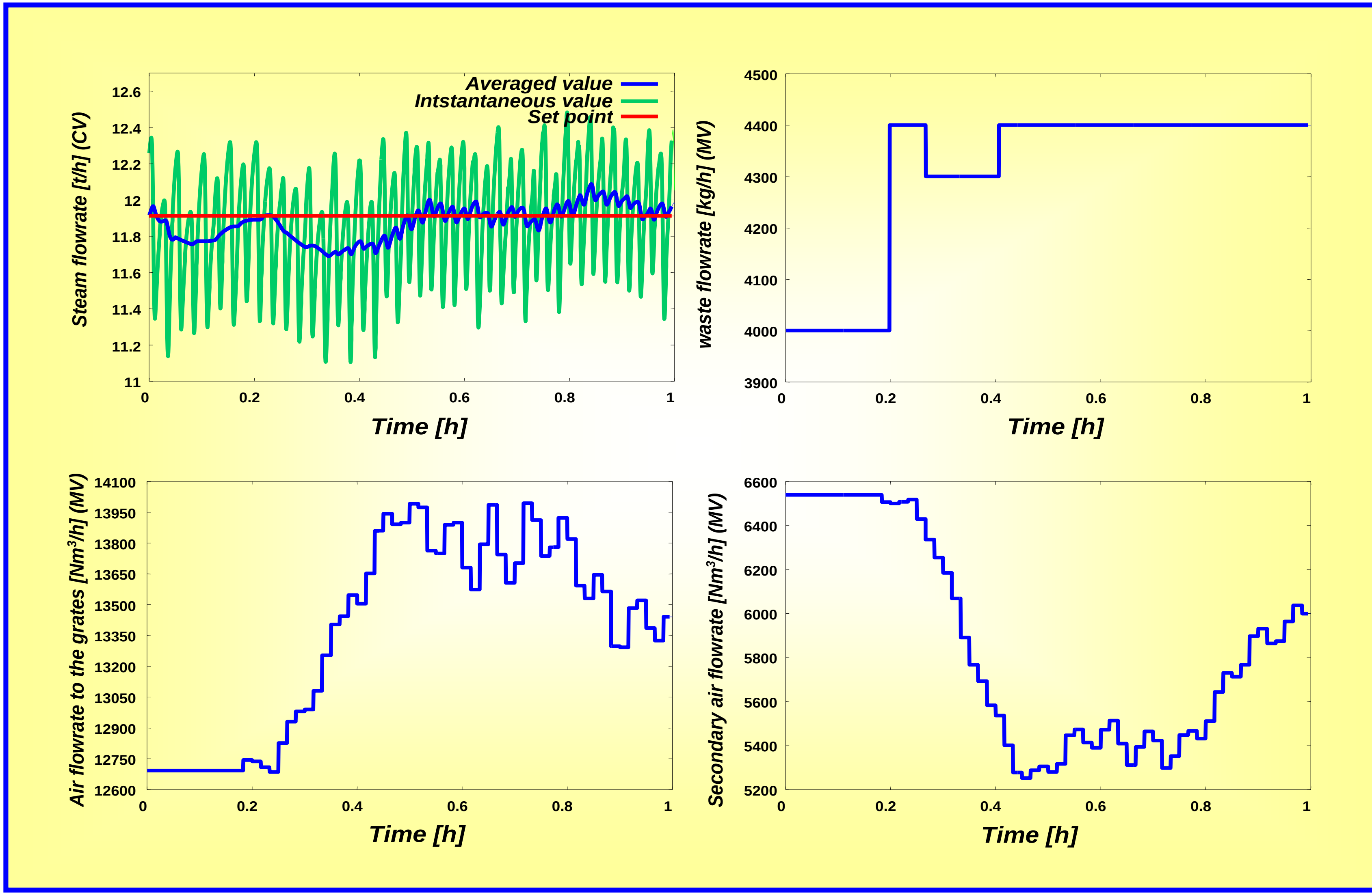
NL MPC – Comparison with PI controller



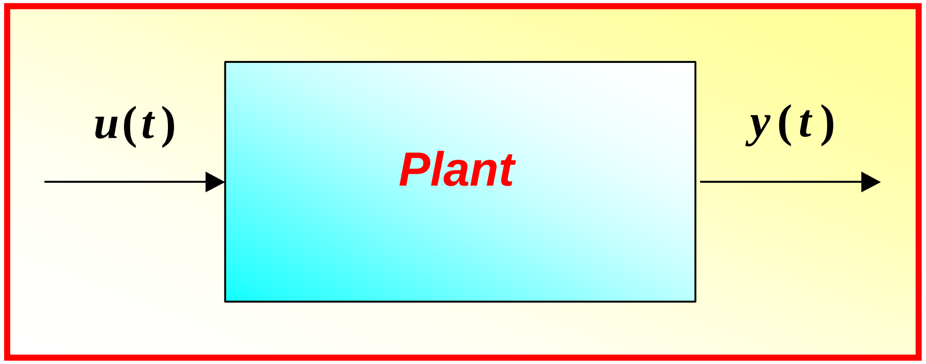
MPC Controller

- Faster control action
- Reduced oscillations of both the controlled and manipulated variables

NL MPC – Regulator problem



Linear MPC – ARX model



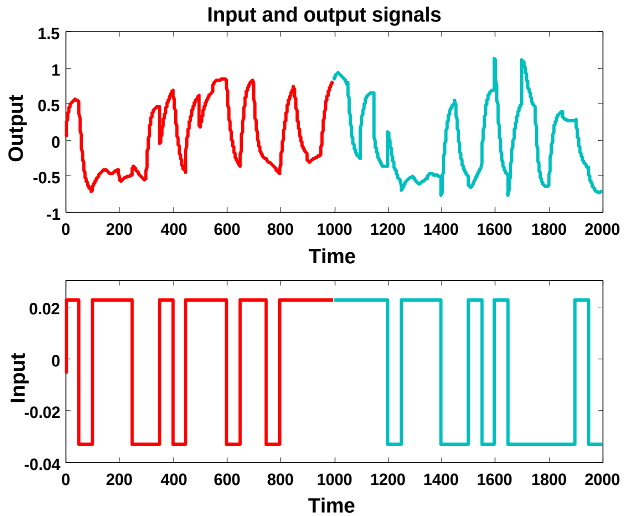
ARX (Auto Regressive Model with Exogenous Input)

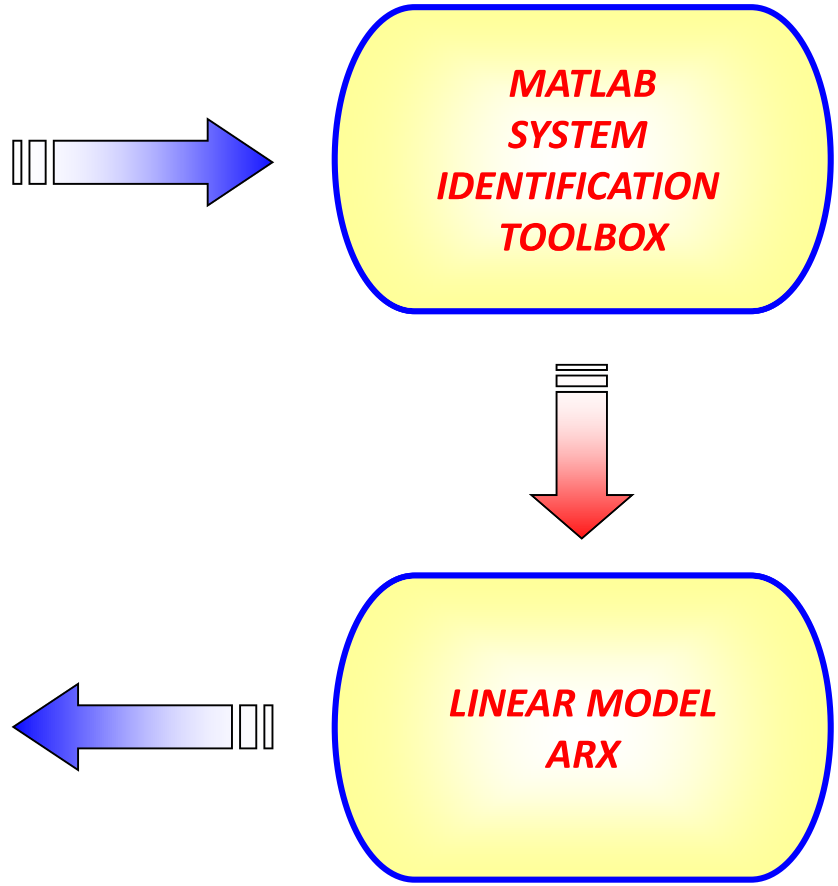
$$\mathbf{y}_k + \mathbf{a}_1 \cdot \mathbf{y}_{k-1} + \mathbf{a}_2 \cdot \mathbf{y}_{k-2} + \cdots + \mathbf{a}_{na} \cdot \mathbf{y}_{k-na} = \mathbf{b}_1 \cdot \mathbf{u}_{k-1} + \mathbf{b}_2 \cdot \mathbf{u}_{k-2} + \cdots + \mathbf{b}_{nb} \cdot \mathbf{u}_{k-nb}$$

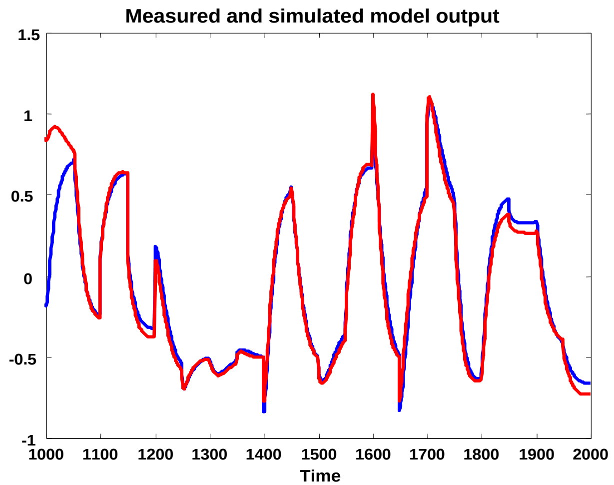
$\mathbf{y}_{k-i}$	**NY *output* vector at time *(k-i)***
$\mathbf{u}_{k-j}$	**NU *input* vector at time *(k-j)***
$\mathbf{a}_i$	**Matrix of NY·NY *output* parameters at time *(k-i)***
$\mathbf{b}_j$	**Matrix of NY·NU *input* parameters at time *(k-j)***
na	**Model order respect to the *outputs***
nb	**Model order respect to the *inputs***

Linear MPC – Identification procedure

PRBS Input (Pseudo Random Binary Sequence)

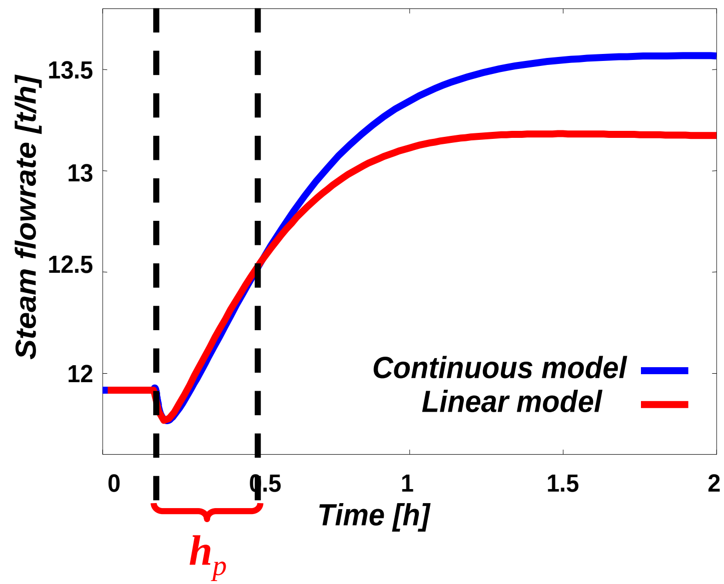






Linear MPC – Models comparison

Similar trends within the prediction horizon

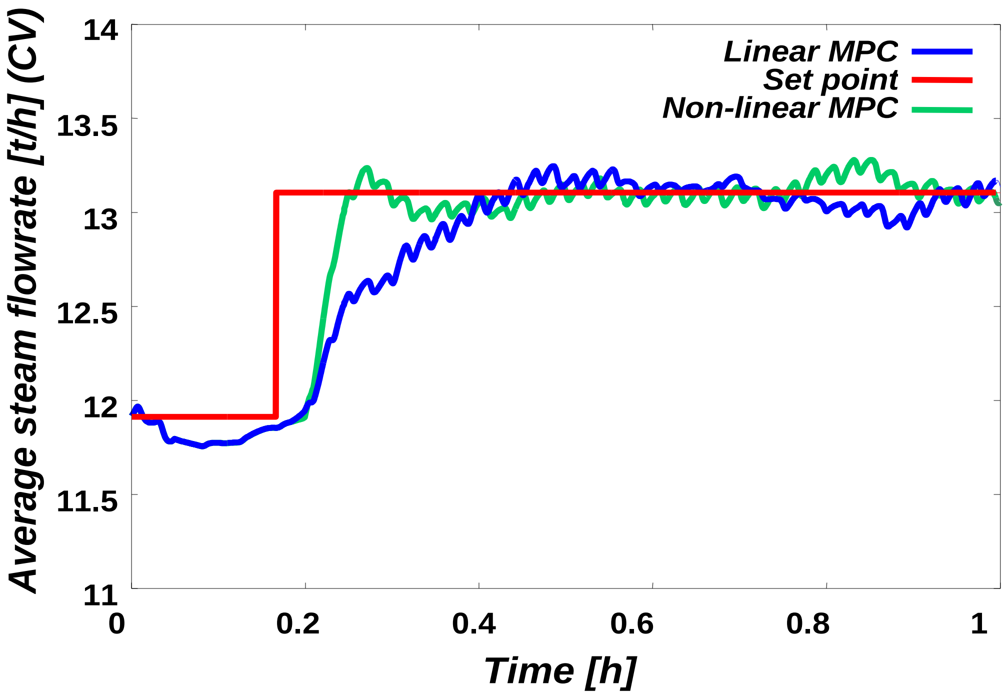


CPU times get significantly reduced

CPU time for a 1 min simulation interval	10% disturbance on the waste flowrate
Simplified continuous model (68 DAE)	0.54s
Linear model (ARX)	4.E-5s

Linear MPC – Servo problem

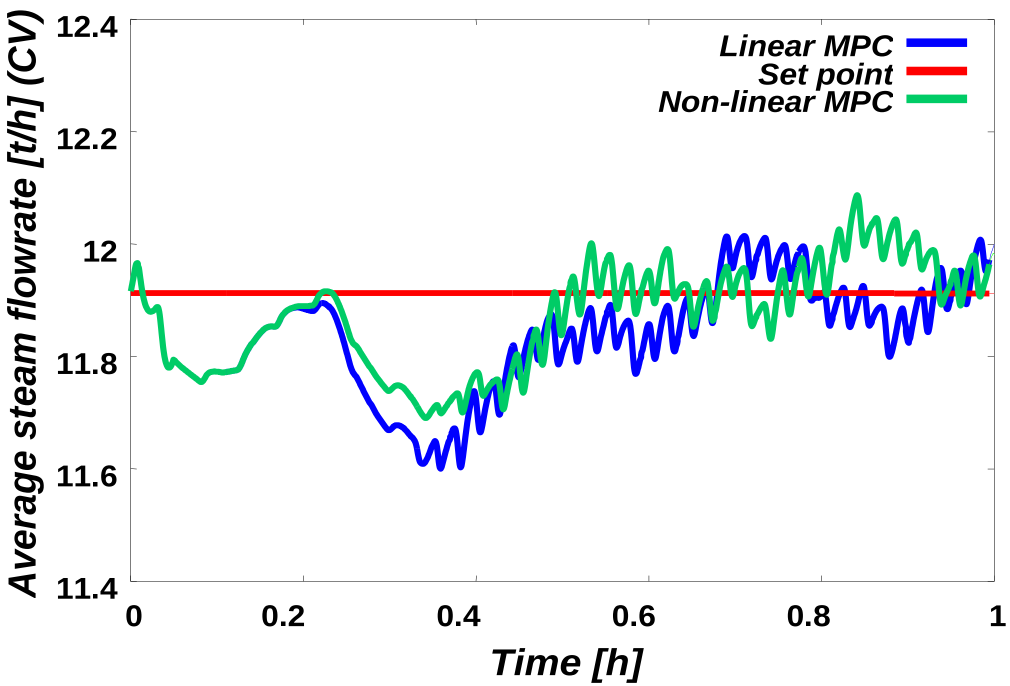
Comparison between linear and non-linear MPC



The non-linear MPC reaches the setpoint faster

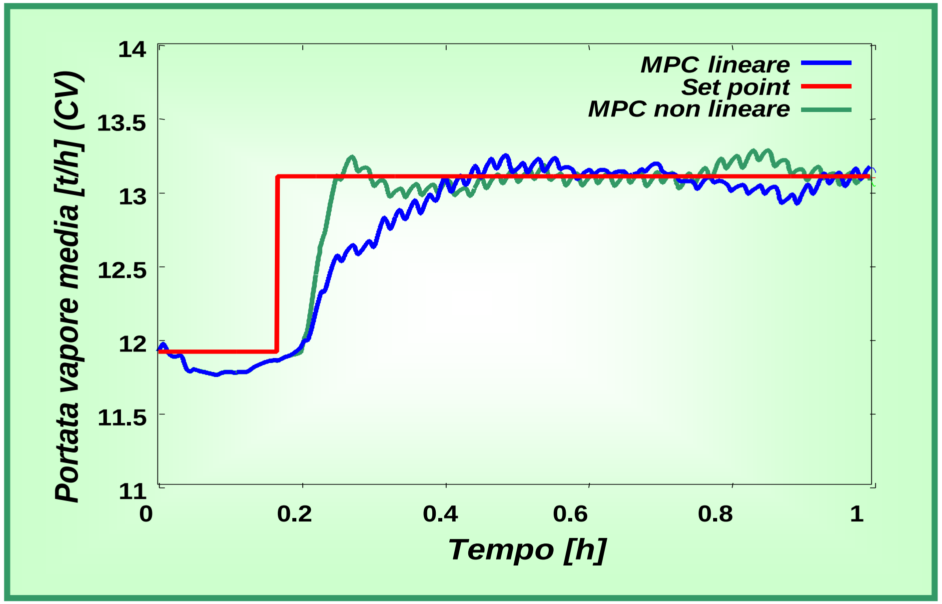
Linear MPC – Regulator problem

Comparison between linear and non-linear MPC



The non-linear MPC shows a smaller deviation from the setpoint

Conclusions



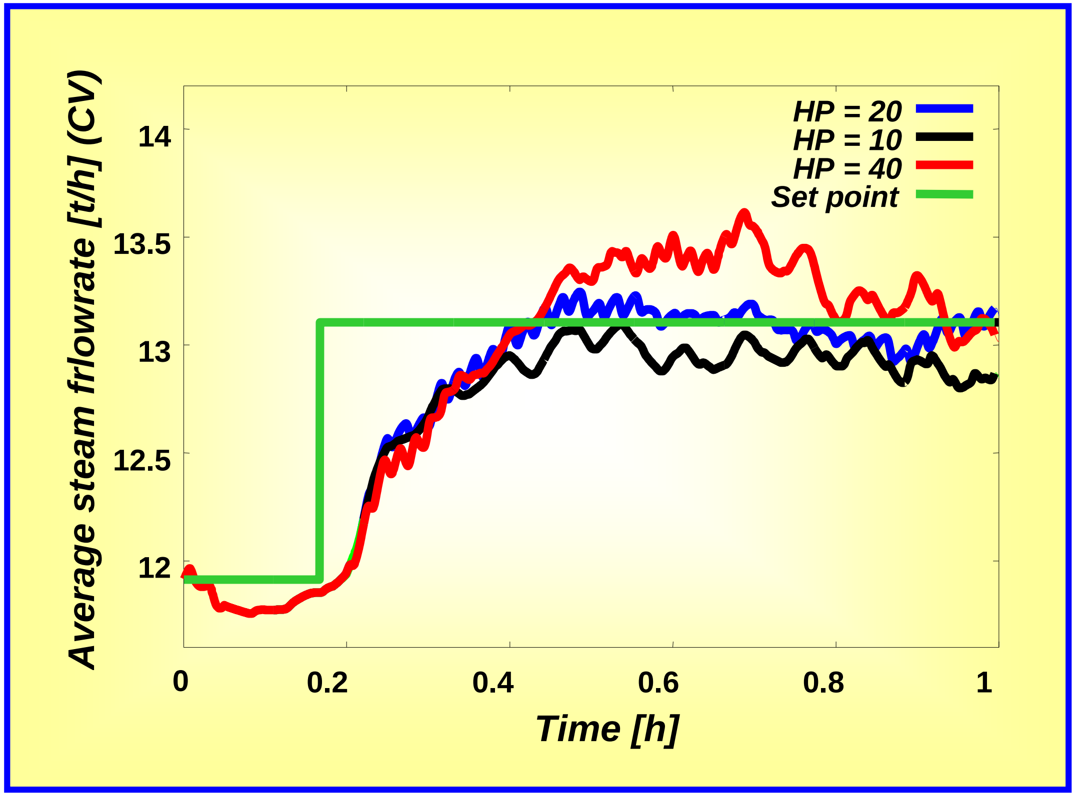
- ***The control efficiency is comparable***
- ***The CPU time is shorter than the control time only for the linear MPC***
- ***Higher robustness for the linear MPC***

	CPU time for a 20 min simulation of the whole plant	**CPU time of the optimization procedure (600 F_{obj} calls)**
Non-linear MPC	0.65 [s]	390. [s]
Linear MPC	7.E-4 [s]	0.45 [s]

Optimization CPU time < t_c

The implementation of the linear MPC on the real plant is viable

Linear MPC – Effect of varying h_p

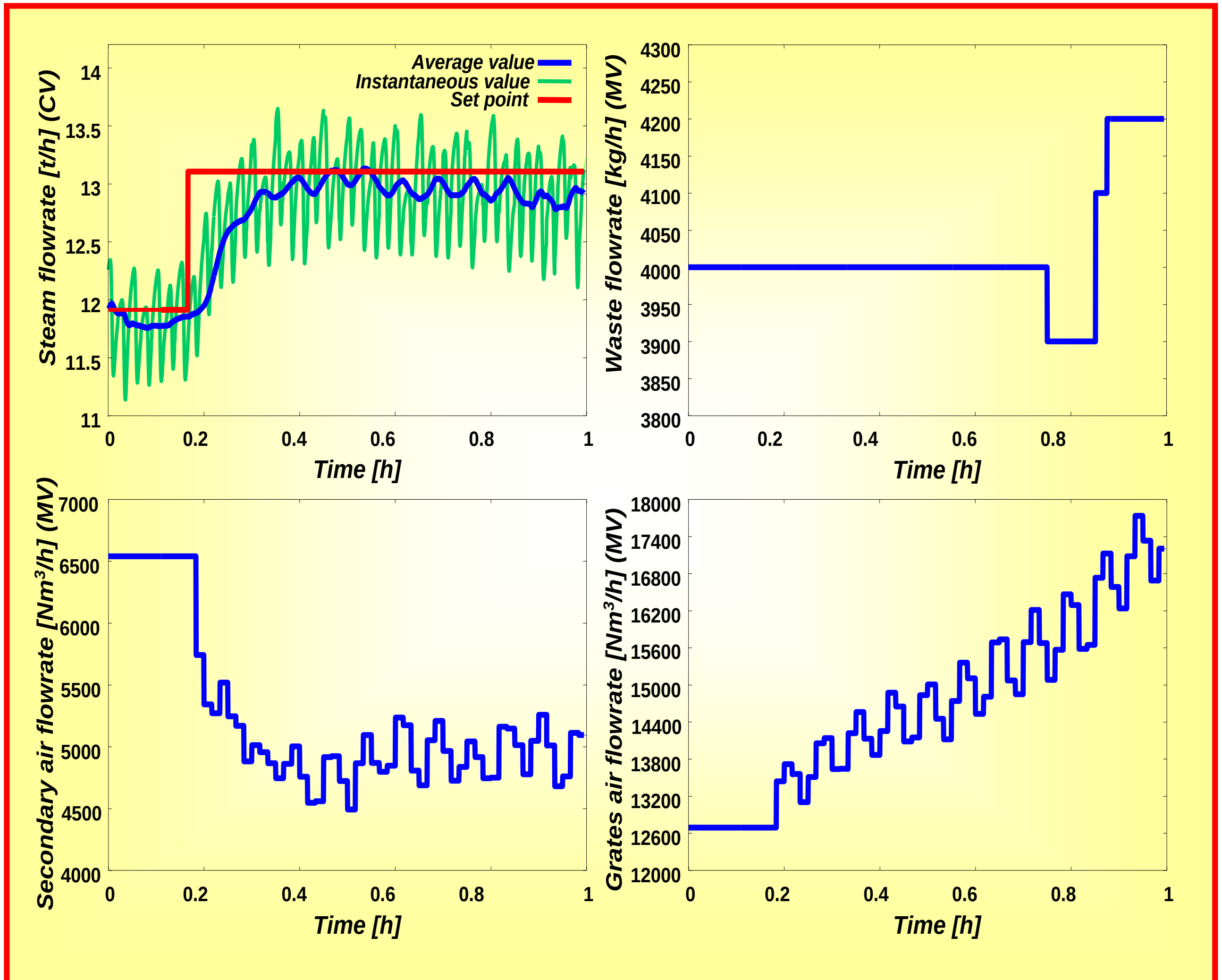


Prediction horizon, h_p:

short: reduced predictive capability

long: the prediction becomes less realistic

Linear MPC – Effect of varying h_c



$h_c = 3$

Control horizon, h_c:

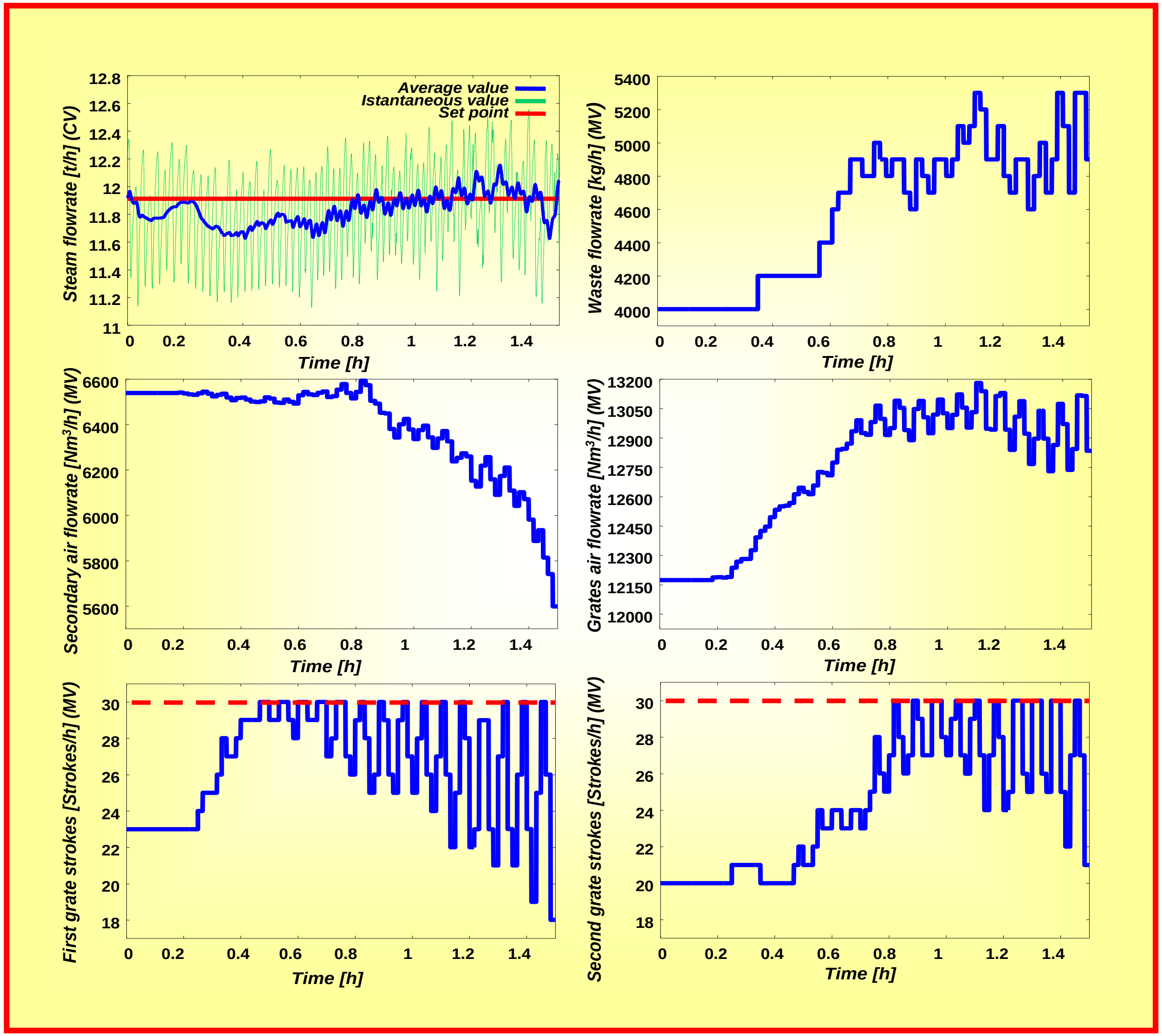
short:
Few degrees of freedom.
Bolder actions from manipulated variables

long:
Longer CPU time, Smoother control action

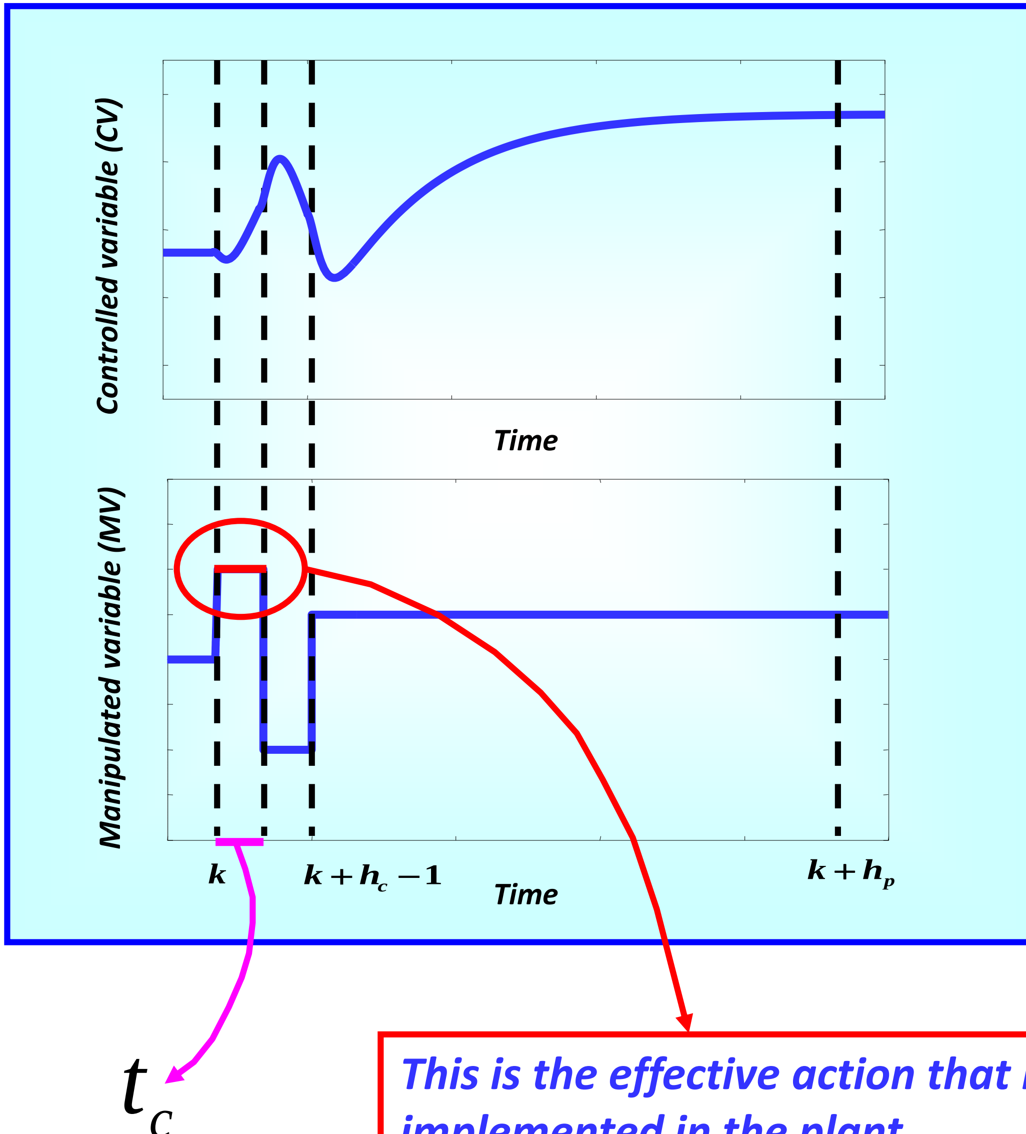
Linear MPC – Ringing

Small weighs (ω_u) on the manipulated variables

The controller becomes nervous.
We can observe a RINGING trend



Linear MPC – Prediction and control horizon



This is the effective action that is implemented in the plant

t_c **Control time:**

Shorter than 1/10 of the characteristic time

$$h_c \le h_p$$

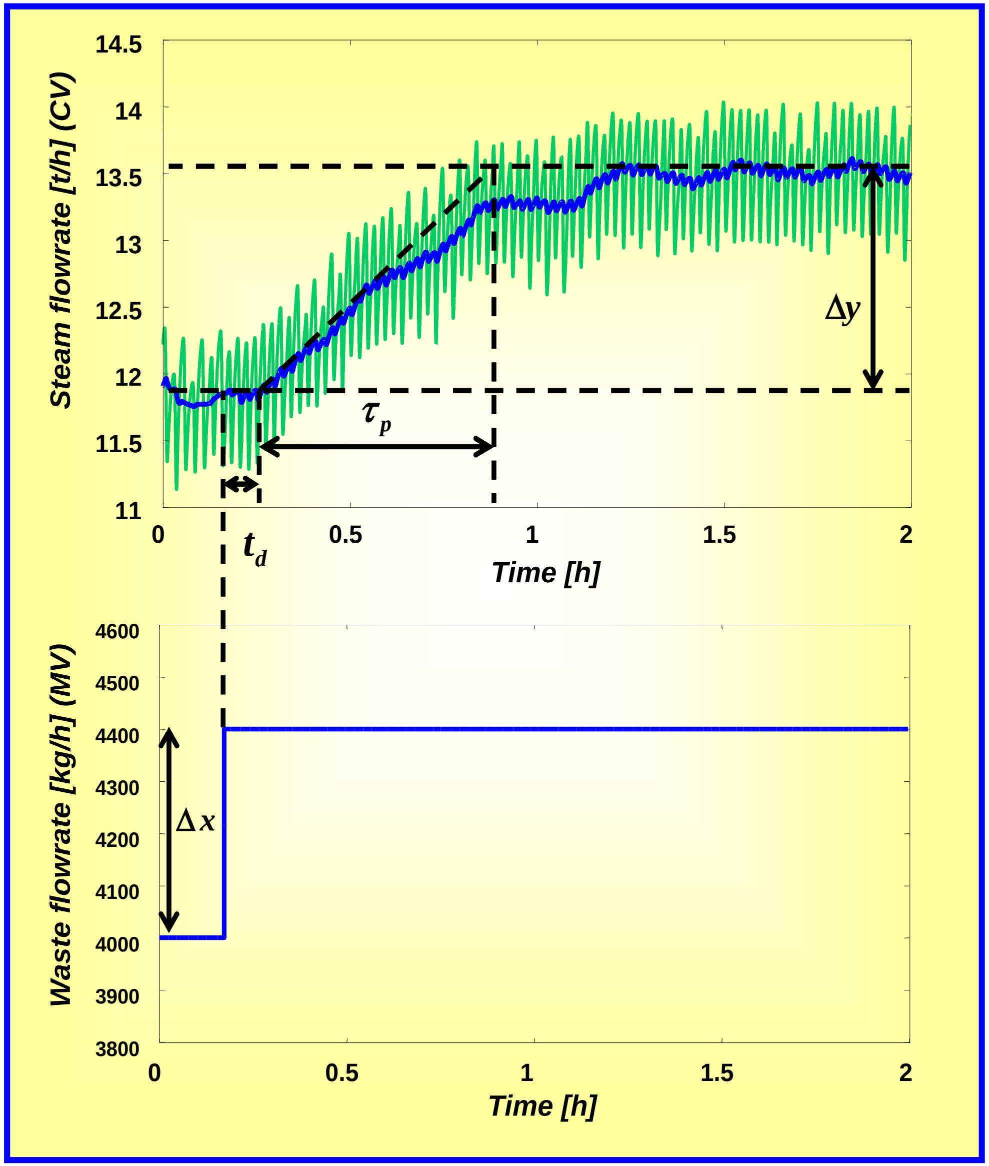
h_p **high:**

- ***High predictive capability***
- ***Significant role played by the model error***

h_c **high :**

- ***High number of d.o.f.***
- ***Smoother control actions***

MPC – Selection of the controller parameters



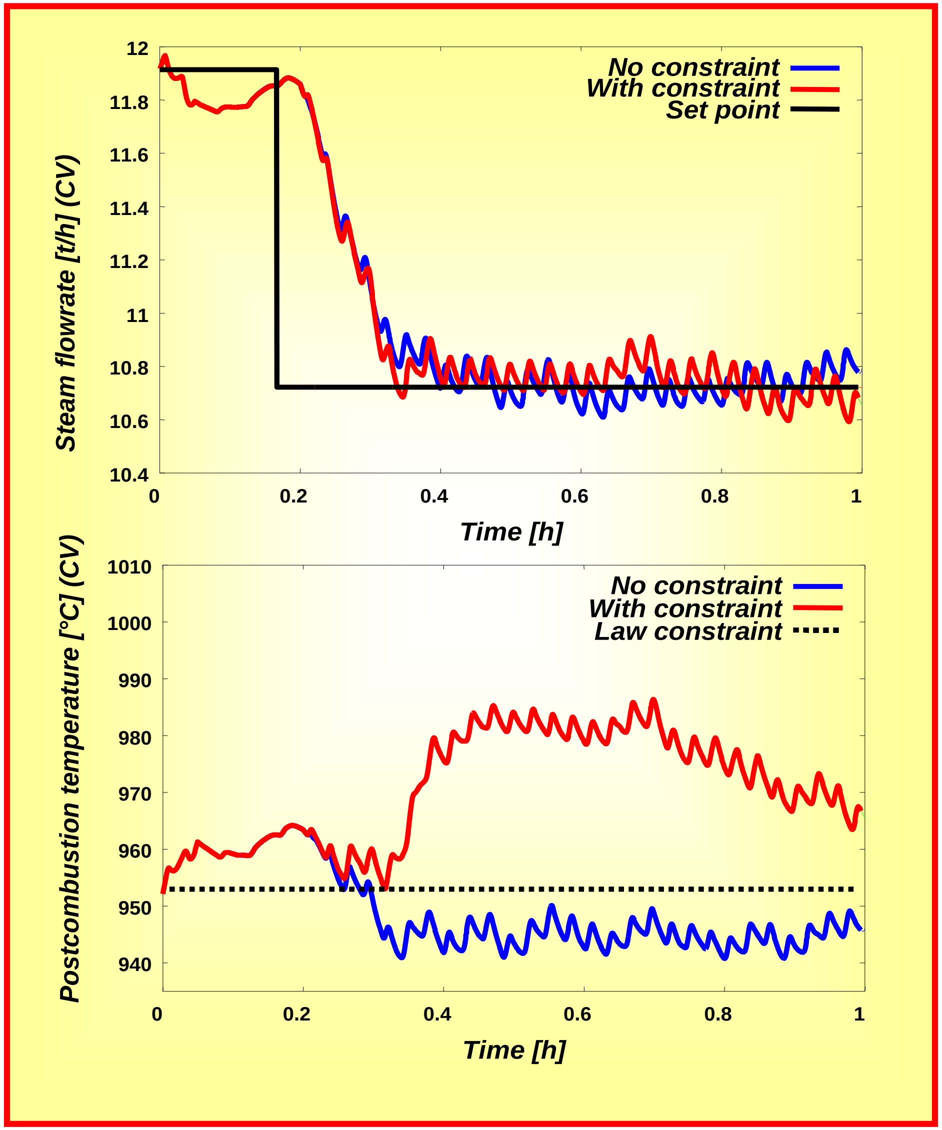
$$\tau_p + t_d \cong 40\ min$$

t_c = 1 min

h_p = 20

h_c = 1

NL MPC – Constraints

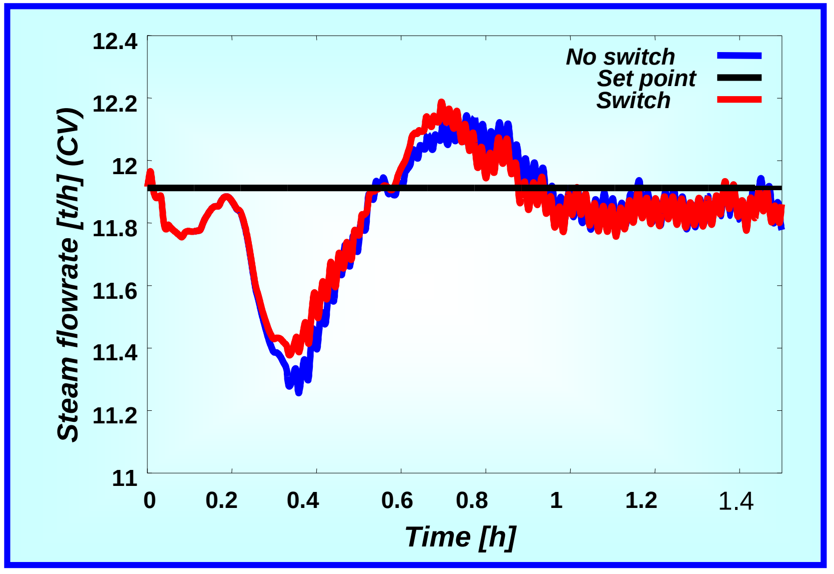


Setpoint step change of –10%

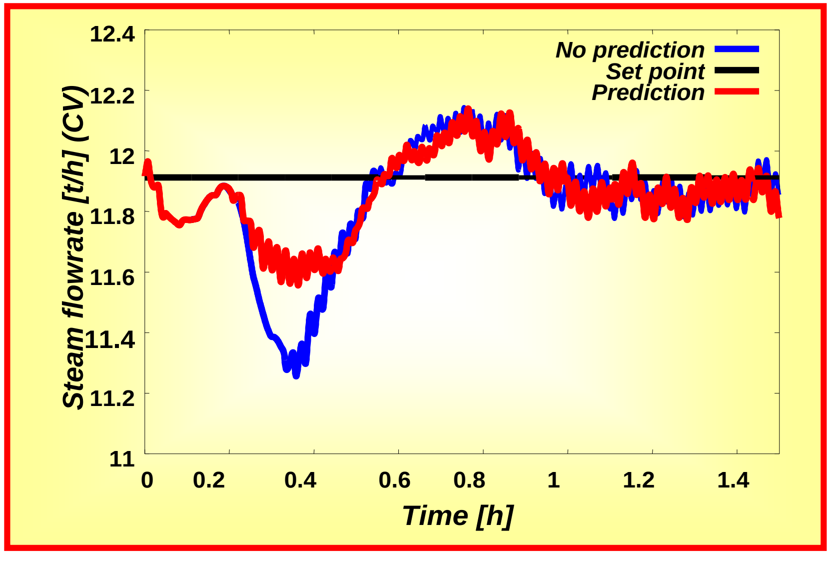
Law constraint on the outlet temperature from the postcombustion chamber:

T > 950 °C

Linear MPC – Heat of combustion



"Switch" between two ARX models identified according to two different waste heats of combustion



The waste heat of combustion is an input value of the ARX model