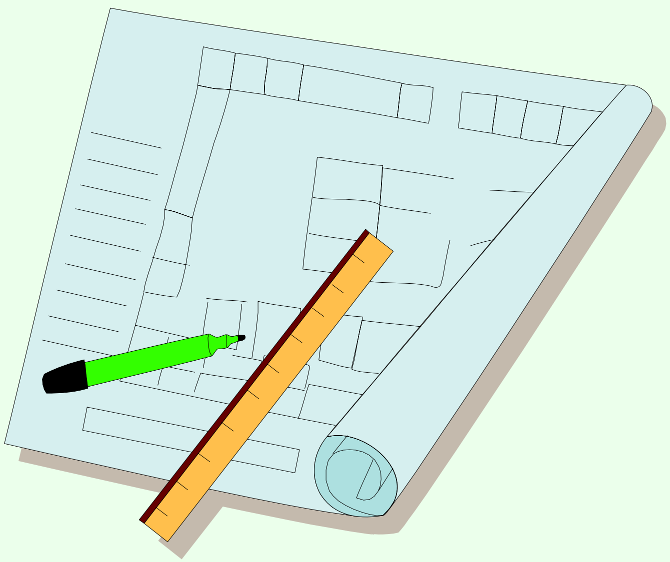
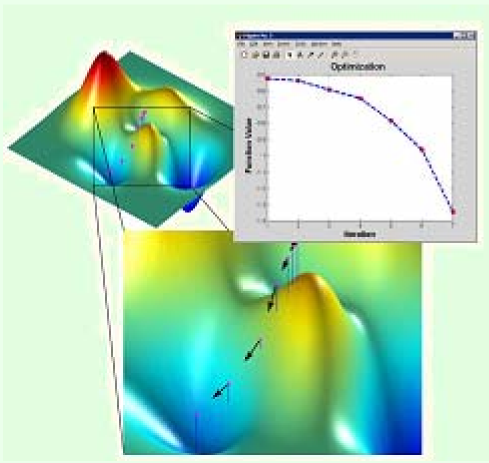
E1

Calcolo numerico – Algoritmi





Metodologie per la risoluzione di problemi

È possibile riassumere in 10 punti sequenziali l'approccio ottimale alla risoluzione di un problema numerico:

1) Comprendere gli obiettivi e le richieste specifiche del problema da risolvere
2) Identificare i dati a disposizione. Raccogliere i dati mancanti. Ricordare che alcuni dei dati a disposizione potrebbero rivelarsi non necessari
3) Identificare la/le incognite da determinare
4) Semplificare, per quanto possibile, il problema con l'obiettivo comunque di identificare correttamente ciò che viene richiesto. Definire in modo esplicito le assunzioni prese nel corso della definizione e della risoluzione del problema
5) Disegnare, se possibile, una schematizzazione del problema o della procedura risolutiva intrapresa e dare un nome alle singole variabili

Metodologie per la risoluzione di problemi

6) Identificare quali principi fondamentali possono essere applicati per la risoluzione del problema

7) Definire il proprio approccio risolutivo e quindi pensare ad approcci alternativi prima di procedere con il calcolo

8) Dare un nome e commentare ogni passo della procedura risolutiva

9) Se si risolve il problema con un programma di calcolo controllare i risultati risolvendo a mano un problema analogo ma semplificato. Effettuare un'analisi di consistenza dimensionale e delle unità di misura. Stampare a video o su file i valori delle variabili intermedie

10) Effettuare infine un'analisi di consistenza dei risultati prodotti. Hanno senso? Rispettano gli ordini di grandezza attesi? Esistono più soluzioni al problema posto? Hanno senso fisico o nascono solo dalla formulazione matematica?

Esercizi

E1.1

Determinare con Matlab™ e verificare con la calcolatrice i risultati delle seguenti operazioni:

1) $8+3\cdot 5$

2) $\dfrac{3^4-5^2}{3(7-5)}-4\dfrac{12-\dfrac{3}{7}}{\dfrac{51}{3}}$

3) $7\cdot 35^{1/4}+9\cdot 12.38^{0.49}$

4) $\sqrt{3+4^3}-\sqrt{3-4^{1/3}}$

Problemi

E1.2

Un cilindro è alto 15 m e ha raggio 8 m. Si desidera costruire un secondo cilindro avente volume maggiore del 20% rispetto al primo ma avente stessa altezza. Identificare le dimensioni del secondo cilindro.

Risoluzione: *applicare i 10 punti presentati in precedenza e risolvere il problema utilizzando come linguaggio di programmazione* Matlab™.

E1.3

Determinare la somma dei primi 50, 100, 1000 e 1,000,000 di numeri interi. Esiste anche una formula più veloce per determinare il valore di:

$$S = \sum_{k=1}^{n} k$$

Determinarla e confrontare i risultati. Costruire una procedura specifica in Matlab™.

Problemi

E1.4

A proposito di "ricette" numeriche… Implementare la sequenza di istruzioni:

```
inizializzare la variabile z con un valore positivo a piacere
porre x pari a: z/2
ripetere per 6 volte:
   porre x pari a: (x+z/x)/2
   visualizzare x
stop
```

- Osservare attentamente il risultato e per via empirica determinare che cosa produce l'algoritmo numerico summenzionato.
- Trasformare la procedura in uno *script* di Matlab™ chiedendo all'utente di introdurre da tastiera il valore della variabile **z** (spunto: cercare nel help il comando **input**)
- Confrontare il risultato della procedura con quello ottenibile per… altra via…

Problemi

E1.5

Costruire una funzione di Matlab in grado di calcolare il fattoriale di un numero. Quindi utilizzarla dalla linea di comando, confrontando i risultati con l'eventuale funzione interna a Matlab™.

Testare la procedura fornendo non solo valori interi positivi ma anche valori reali o negativi.

Incrementare la *robustezza* della funzione implementata al fine di essere in grado di *resistere* all'utilizzo di un utente sprovveduto, distratto, inesperto, malizioso…

E1.6

Determinare quanti anni sono necessari per accumulare almeno la somma di € 30,000 avendo depositato in banca in una sola soluzione € 10,000, potendo contare su di un interesse creditore, al netto di tutte le spese, pari al 3.51%.

Diagrammare l'andamento del deposito bancario negli anni.

Modello Preda — Predatore di Volterra Lotka

Two Species Models

The models we have discussed so far (Malthus and Logistic) are *single species* models. Many of the most interesting dynamics in the biological world have to do with interactions between species. Mathematical models which incorporate these interactions are required if we hope to simulate these dynamics. One of the first models to incorporate interactions between predators and prey was proposed in 1925 by the American biophysicist Alfred Lotka and the Italian mathematician Vito Volterra.

Lotka-Volterra

The Lotka-Volterra model describes interactions between two species in an ecosystem, a predator and a prey. This represents our first multi-species model. Since we are considering two species, the model will involve two equations, one which describes how the prey population changes and the second which describes how the predator population changes. For concreteness let us assume that the prey in our model are rabbits, and that the predators are foxes. If we let $R(n)$ and $F(n)$ represent the number of rabbits and foxes, respectively, that are alive at time period n, then the Lotka-Volterra model is:

$$R(n+1) = R(n) + a*R(n) - b*R(n)*F(n)$$
$$F(n+1) = F(n) + e*b*R(n)*F(n) - c*F(n)$$

where the parameters are defined by:

- *a* is the natural growth rate of rabbits in the absence of predation,
- *c* is the natural death rate of foxes in the absence of food (rabbits),
- *b* is the death rate per encounter of rabbits due to predation,
- *e* is the efficiency of turning predated rabbits into foxes.

Modello Preda — Predatore di Volterra Lotka

The Stella model representing the Lotka-Volterra model will be slightly more complex than the single species models we've dealt with before. The main difference is that our model will have two stocks (reservoirs), one for each species. Each species will have its own birth and death rates. In addition, the Lotka-Volterra model involves four parameters rather than two. All told, the Stella representation of the Lotka-Volterra model will use two stocks, four flows, four converters and many connectors.

Exercises

1.Split the rabbit's difference equation into the births part and the deaths part.

2.Do the same for the fox's equation.

3.Using the following parameter values, write down the difference equations for the Lotka-Volterra model and find all equilibrium points. This will involve solving two equations for two unknowns (namely *R(*)* and *F(*)*). **HINT:** this model produces two steady states, one of which should be unsurprising.

- $a = 0.04$
- $b = 0.0005$
- $c = 0.2$
- $e = 0.1$
- ***Optional:*** *Try to find expressions for the Lotka-Volterra steady states in terms of the parameters. In other words, try to find formulas for R(*) and F(*) without plugging in specific values for the parameters.*
- *Create a Stella model for the Lotka-Volterra model. Use the parameter values given above as values for the four converters in your model. Try various initial conditions for the rabbits and fox populations; choose some to be near the equilibria you determined above, and have some be far away. Use different running times. Which equilibrium is stable, unstable?*

Modello Preda — Predatore di Volterra Lotka

•Try both the usual time series graph and the scatter graph to examine the model output. A scatter plot of rabbit versus fox population is particularly interesting. To produce such a graph pull down the graph icon, place it somewhere in the model, double click on the graph when it appears and select the scatter plot option. This will require you to choose two quantities to plot. Pick the rabbit and fox populations. You should get some interesting pictures if you let the model run long enough.

•**Analysis of Lotka-Volterra**

•The Lotka-Volterra model is one of the earliest predator-prey models to be based on sound mathematical principles. It forms the basis of many models used today in the analysis of population dynamics. Unfortunately, in its original form Lotka-Volterra has some significant problems. As you may have noted in your experiments, neither equilibrium point is stable. Instead the predator and prey populations seem to cycle endlessly without settling down quickly. It can be shown (see any undergraduate differential equations book for details) that this behavior will be observed for any set of values of the model's four parameters. While this cycling has been observed in nature, it is not overwhelmingly common. It appears that Lotka-Volterra by itself is not sufficient to model many predator-prey systems. Context specific information must be added.

•**A Modification of Lotka-Volterra**

•A slight modification of Lotka-Volterra creates a stable equilibrium in the model. The idea is to use a logistic model for each species in the absence of the other. The resulting equations are

R(n+1) = R(n) + c*R(n)*(1-R(n)/KR) - d*R(n)*F(n)

F(n+1) = F(n) + a*F(n)*(1-F(n)/KF) + b*F(n)*R(n)

Modello Preda — Predatore di Volterra Lotka

where KR and KF are the carrying capacities for rabbits and foxes, respectively. The other parameters, a, b, c and d have new interpretations.

•Before these equations can be used in STELLA they must be split up into their inflow and outflow components (or a bidirectional valve can be used). Rewriting these equations in the separated form gives us:

R(n+1) = R(n) + c*R(n) - (c*R(n)*R(n)/KR + d*R(n)*F(n))

F(n+1) = F(n) + (a*F(n) + b*F(n)*R(n)) - a*F(n)*F(n)/KF

Supercomputer

HIGHLIGHTS FROM THE TOP 10

•The Earth Simulator, built by NEC, remains the unchallenged #1.

•ASCI Q at Los Alamos has now been measured with 13.88 TF/s and is the second system to exceed the 10 TF/s mark.

•The upgraded IBM SP system at NERSC/Lawrence Berkeley Lab at position 5 was recorded with a new IBM record of 7.3 TF/s. This Linpack number even exceeds the old measurement of the larger ASCI White system at Lawrence Livermore (#4), with otherwise identical architecture.

•With this new Linpack implementation, ASCI White could achieve a better result, but the system was unavailable for testing.

•Therefore, the Intel Xeon-based MCR cluster at Lawrence Livermore Lab outranked ASCI White and achieved the highsets position for any cluster yet (#3). It uses a Quadrics interconnect.

•Fujitsu's PrimePower HPC2500 system at the National Aerospace Laboratory of Japan is the largest new Japanese system at position 7.

•#8 at Pacific Northwest National Lab is the highest-ranked Itanium-based system. It is produced by Hewlett-Packard.

•Seven of the TOP 10 systems are installed in the US, two in Japan, and one in France.

•The performance of the #10 system came in with 3.98 TF/s, just shy of the 4 TF/s mark.

http://www.top500.org

Supercomputer

General highlights from the Top 500:

•The number of systems exceeding the 1 TFflop/s mark on the Linpack is now 59, up from to 47 just six month ago. Already 157 system report a peak performance above 1 TF/s.

•The last system on the list with 245.1 GF/s was listed at position 285 in the last TOP500 just six month ago. This indicates a turnover rate well above average.

•Total accumulated performance is 375 TFlop/s, compared to 293 TFlop/s six month ago.

•Entry level is now 245.1 GF/s, compared to 195.8 GF/s six month ago.

•The entry point for the top 100 moved from 558 GF/s to 708 GF/s.

•A total of 119 system, up from 56 six month ago, are now using Intel processors.

•149 systems are now labeld as cluster, up from 93.

•23 of these clusters are labeled as self-made, up from 14.

•IBM is still leading the list with respect to the installed performance and increased its share from 31.8% to 34.9%. HP is second in performance with 24.1%, up from 22.1%, and NEC is third with 11.7%.

•With respect to the number of systems, Hewlett-Packard stayed barely ahead of IBM by only one system. HP is at 159 systems (up from 137) and IBM at 158 systems (up from 131). installed (HP 137 and IBM 131). SGI is now third with 54 systems.

•No other manufacturer is able to capture more than 6% in any category.

•The new Cray X1 system makes its first show in the list with 10 installations.

•A total of 12 cluster integrators showed up in the list. Most of them feature a single system on the list, with the notable exceptions of Linux Networx (6 systems), Atipa Technology (3 systems), and Megware (2 systems).

Supercomputer

Introduction

The Earth Simulator (ES) is a project of Japanese agencies NASDA, JAERI and JAMSTEC to develop a 40 TFLOPS system for climate modeling.

The ES site is a new location in an industrial area of Yokohama, an hour drive west of Tokyo. The facility became operational in late 2001, and claimed first spot in current Top 500 list. In spite of a public 5-year development period leading to that event, the US supercomputing community was caught by surprise.

Hardware

The ES is based on:

- 5,120 (640 8-way nodes) 500 MHz NEC CPUs
- 8 GFLOPS per CPU (41 TFLOPS total)
- 2 GB (4 512 MB FPLRAM modules) per CPU (10 TB total)
- shared memory inside the node
- 640 × 640 crossbar switch between the nodes
- 16 GB/s inter-node bandwidth
- 20 kVA power consumption per node (**circa 13 MW totali**)

Supercomputer

The vector CPU is made using 0.15 µm CMOS process, and is a descendant (same speed, smaller process) of the NEC SX-5 CPU. The machine runs a version of the Super-UX UNIX-based OS. OpenMP parallel directives are used within a node, and MPI-2 or HPF must be used across multiple nodes, necessitating a dual-level parallel implementation. In fact this can be considered a three-level parallel system, if single-CPU vectorization is taken into account; however, vectorization is largely automatic. Still, an optimized code will need to employ MPI-2 at the subdomain level, OpenMP at the loop level, and vectorization directives at the instruction level all at once.

Physical

The CPUs are housed in 320 cabinets, 2 8-CPU nodes per cabinet. The cabinets (blue) are organized in a ring around the interconnect, which is housed in another 65 cabinets (green). Another layer of the circle is formed by disk array cabinets (white). The whole thing occupies a building 65 m long and 50 m wide. Activity on the nodes is signalled by a bright green beacon at the top of the cabinet, and if a fault occurs, a similar red light turns on. Switch cabinets also have green and red signaling lights for various types of communication events.

The machine room sits at approximately 4th floor level. The 3rd floor level is taken by hundreds of kilometers of copper cabling, and the lower floors house the air conditioning and electrical equipment. The structure is enclosed in a cooling shell, with the air pumped from underneath through the cabinets, and collected to the two long sides of the building. The aeroshell gives the building its "pumped-up" appearance. The machine room is electromagnetically shielded to prevent interference from nearby expressway and rail. Even the halogen light sources are outside the shield, and the light is distributed by a grid of scattering pipes under the ceiling. The entire structure is mechanically isolated from the surroundings, suspended in order to make it less prone to earthquake damage. All attachments (power, cooling, access walkways) are flexible.\

http://www.es.jamstec.go.jp/esc/eng/ESC/index.html

Our mission is

To build a harmonious relationship between the Earth and human beings, the Earth Simulator Center will endeavor to achieve the maximum benefit of the Earth Simulator' s capabilities, and will pursue challenges in various areas of research and development. Through collaboration with various national-related agencies and industries and with the support of the Japanese nation, the Earth Simulator Project is dedicated to serving society.

Basic Principles

1. Quantitative prediction and assessment of variations of the atmosphere, ocean and solid earth.
2. Production of reliable data to protect human lives and properties from natural disasters and environmental destruction.
3. Contribution to symbiotic relationship of human activities with nature.
4. Promotion of innovative and epoch-making simulation in any fields such as industry, bioscience, energy and so on.


The Earth Simulator Center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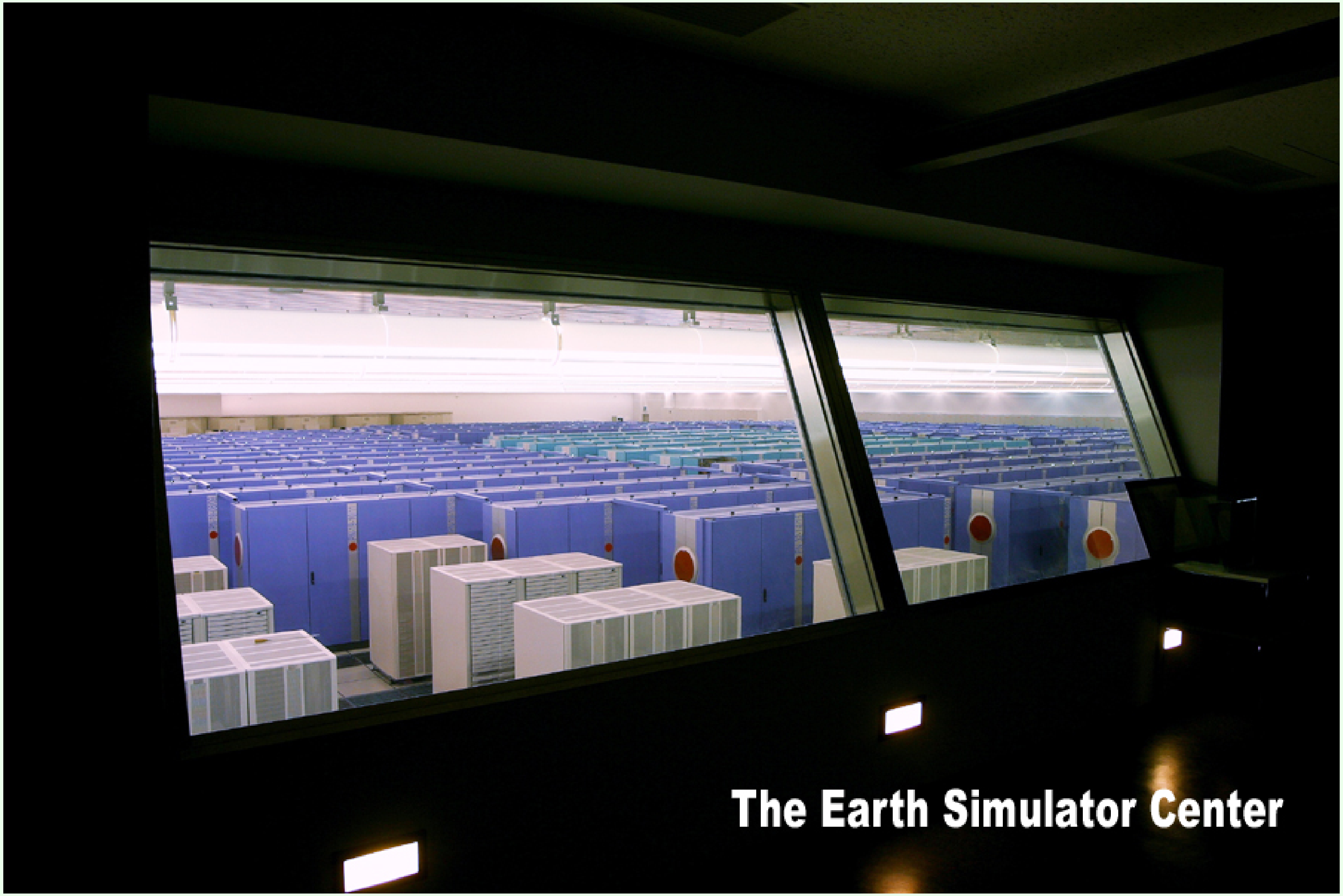
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Riferimenti Web

http://www.stolaf.edu/people/mckelvey/envision.dir/predprey.dir/predprey.html

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