Model Predictive Control

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Lecture 6 of "Dynamics and Control of Chemical Processes" – Master Degree in Chemical Engineering

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Introduction

- Industrial processes are characterized by the following control problems:

  1. The **process** is almost usually **multivariate**
     - several controlled variables: $y_1, y_2, \ldots, y_n$
     - several manipulated variables: $u_1, u_2, \ldots, u_m$
     - several disturbance variables: $d_1, d_2, \ldots, d_k$
  2. **Complex dynamic** behaviour:
     - *time delays* due to the inertia of the system (either material or energetic), mass flow in the pipes, long measuring times
     - Inverse response
     - possible instability at open-loop

Introduction

3. Intrinsic **nonlinearity** of the system
4. **Operative constraints** that are quite dissimilar and complex
   - constraints on the input and output variables
   - constraints on the changing rate of the input variables
   - Constraints on the optimal value of the input variables (*e.g.*, economic value)
   - process and law constraints
   - soft and hard constraints

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Model based control

An **ideal control system** should be:

- Multivariable and capable of managing:
  - time delays,
  - inverse response,
  - process and law constraints,
  - measurable and non-measurable disturbances
- Minimize the control effort
- Able of inferring the unmeasured/unmeasurable variables from the measured ones
- Robust respect to the modeling errors/simplifications and the noise of the measured variables
- Able to manage both the *startups* and *shutdowns* (either programmed or emergency) as well as the steady-state conditions

Model based control

- The availability of **dynamic numerical models** of:
  - chemical/industrial processes,
  - unit operations
  - process units,
  - plant subsections
  - …

  allows **forecasting the response** of the simulated plant/process to possible **disturbances** and **manipulated variables**.

- The availability of such **dynamic numerical models** paves the way to the so-called: **model based control**.
- The model of the process can be *used* to forecast the system response to a set of control actions originated by modifying a suitable set of manipulated variables.

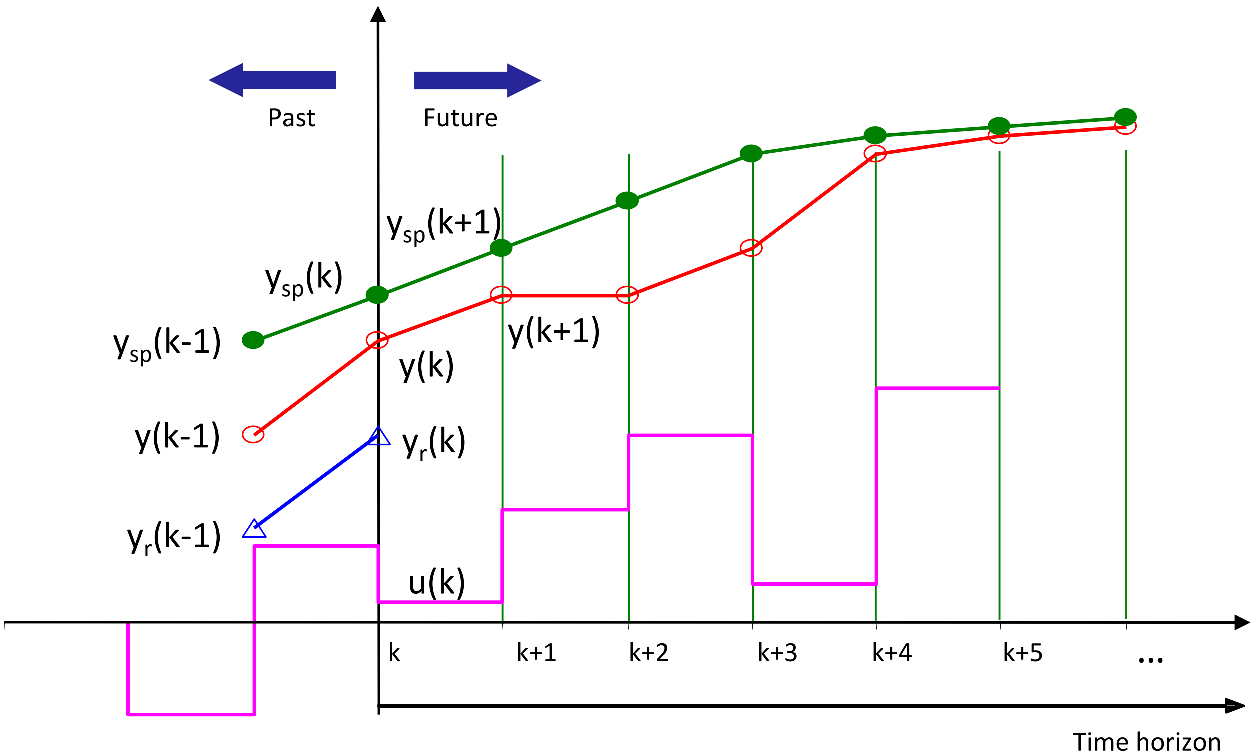
Model based control

- We are going to answer the following question:

  What is the response of the system to a modification of the manipulated variables?

- More specifically, we can imagine to deploy an optimizing procedure that looks for the best response of the system subject to the manipulation of the process variables.
- According to the <u>most simplified approach</u>, we have:
  - the control specifications, *i.e.* the setpoint
  - the objective function that measures the distance of the controlled variable from the setpoint
  - the dynamic model of the system usually described by a DAE system, which plays the role of the equality constraints
  - the manipulated variables that are the degrees of freedom of the optimization problem

Model predictive control



y_{sp} = y set point (setpoint **trajectory**)

y = y model response

y_r = y real, measured response

u = manipulated variable

MPC features

- The system follows a specified trajectory → optimal setpoint trajectory, y_{sp}
- The model is called to produce a prediction, y, of the real response of the system.
- We have:
  - response in the future: $y(k+1)$, $y(k+2)$, $y(k+3)$, …
  - respect to past real inputs: $u(k)$, $u(k-1)$, $u(k-2)$, …
  - respect to future manipulated inputs: $u(k+1)$, $u(k+2)$, …
- The numerical model of the process to be controlled is *used* to evaluate a sequence of control actions that optimize an objective function to:
  - Minimize the system response y respect to the optimal set-point trajectory, y_{sp}
  - Minimize the control effort

MPC features

- Since the model is a simplified representation of the real system, it is intrinsically not perfect. This means that there is a discrepancy between the real system and the modeled one.
- The present error ε_k between the real system and the model is:

$$\varepsilon_k = y_r(k) - y(k)$$

- This error is kept constant and it is used for future forecasts.

MPC mathematical formulation

$$\min_{\mathbf{u}(k),\mathbf{u}(k+1),\ldots,\mathbf{u}(k+h_c-1)} \left\{ \sum_{j=k+1}^{k+h_p} \left[\boldsymbol{\omega}_y \mathbf{e}_y^2(j) + \mathbf{PF}_y(j) \right] + \sum_{i=k}^{k+h_p-1} \left[\boldsymbol{\omega}_u \Delta\mathbf{u}^2(i) + \mathbf{PF}_u(i) \right] + \sum_{l=k}^{k+h_p-1} \boldsymbol{\omega}_T \boldsymbol{\delta}\mathbf{u}_T^2(l) \right\}$$

$$\mathbf{e}_y(j) = \frac{\left[\mathbf{y}(j) + \boldsymbol{\delta}_y(k)\right] - \mathbf{y}_{sp}(j)}{\mathbf{y}_{sp}(j)} \qquad \boldsymbol{\delta}_y(k) = \mathbf{y}_r(k) - \mathbf{y}(k) = \mathbf{y}_{real}(k) - \mathbf{y}_{model}(k)$$

$$\mathbf{PF}_y(j) = \left\{ Max\left[\mathbf{0}, \frac{\mathbf{y}(j) - \mathbf{y}_{MAX}}{\mathbf{y}_{MAX}} \right] \right\}^2 + \left\{ Min\left[\mathbf{0}, \frac{\mathbf{y}(j) - \mathbf{y}_{MIN}}{\mathbf{y}_{MIN}} \right] \right\}^2$$

$$\Delta\mathbf{u}(i) = \frac{\mathbf{u}(i) - \mathbf{u}(i-1)}{\mathbf{u}(i-1)} \qquad \Delta\mathbf{u}_{MIN} \le \Delta\mathbf{u}(i) \le \Delta\mathbf{u}_{MAX}$$

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$$\boldsymbol{\delta}\mathbf{u}_T(l) = \mathbf{u}(l) - \mathbf{u}_T(l)$$

$$s.t. \begin{cases} \mathbf{h}(\mathbf{y}, \mathbf{u}, \mathbf{d}, t) = \mathbf{0} \\ \mathbf{f}(\mathbf{y}', \mathbf{y}, \mathbf{u}, \mathbf{d}, t) = \mathbf{0} \end{cases}$$

MPC mathematical formulation

$$\min_{\mathbf{u}(k),\mathbf{u}(k+1),\ldots,\mathbf{u}(k+h_c-1)} \left\{ \ldots \right\}$$

MPC mathematical formulation

$$\min_{\mathbf{u}(k),\mathbf{u}(k+1),\ldots,\mathbf{u}(k+h_c-1)} \left\{ \sum_{j=k+1}^{k+h_p} \left[\boldsymbol{\omega}_y \mathbf{e}_y^2(j) + \mathbf{PF}_y(j) \right] + \ldots \right\}$$

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MPC mathematical formulation

$$\min_{\mathbf{u}(k),\mathbf{u}(k+1),\ldots,\mathbf{u}(k+h_c-1)} \left\{ \ldots + \sum_{i=k}^{k+h_p-1} \left[\boldsymbol{\omega}_u \Delta\mathbf{u}^2(i) + \mathbf{PF}_u(i) \right] + \ldots \right\}$$

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In general, $\Delta\mathbf{u}_{MIN}$ is negative

MPC mathematical formulation

$$\min_{\mathbf{u}(k),\mathbf{u}(k+1),\ldots,\mathbf{u}(k+h_c-1)} \left\{ \ldots + \ldots + \sum_{l=k}^{k+h_p-1} \boldsymbol{\omega}_T \boldsymbol{\delta}\mathbf{u}_T^2(l) \right\}$$

$$\boldsymbol{\delta}\mathbf{u}_T(l) = \mathbf{u}(l) - \mathbf{u}_T(l)$$

T = target.

It can be the same optimal value of the steady state conditions for the manipulated variables (*e.g.*, nominal operating conditions).

MPC mathematical formulation

$$\min_{\mathbf{u}(k),\mathbf{u}(k+1),\ldots,\mathbf{u}(k+h_c-1)} \left\{ \sum_{j=k+1}^{k+h_p} \left[\boldsymbol{\omega}_y \mathbf{e}_y^2(j) + \mathbf{PF}_y(j) \right] + \sum_{i=k}^{k+h_p-1} \left[\boldsymbol{\omega}_u \Delta\mathbf{u}^2(i) + \mathbf{PF}_u(i) \right] + \sum_{l=k}^{k+h_p-1} \boldsymbol{\omega}_T \boldsymbol{\delta}\mathbf{u}_T^2(l) \right\}$$

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Critical elements of the MPC

$$h_c$$

$$h_p$$

$$\omega_y$$

$$\omega_u$$

$$\omega_T$$

$$\delta_y\left(k\right)$$

$$y = y_{model}$$

$$\Delta u$$

$$u_T$$

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